केंद्रीय विद्यालय संगठन क्षेत्रीय कार्यालय एर्णाकुलम

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केन्द्रीय विद्यालय संगठन

MINIMUM LEARNING MATERIAL

MATHEMATICS- XII

Session 2022-23

	RELATIONS AND FUNCTIONS	
	MCQ (1 MARK EACH)	
1	Let R be a relation on the set N of Natural numbers defined by nRm if n divides m. Then R is (a) Reflexive and symmetric (b) Transitive and Symmetric (c) Equivalence relation (d) Reflexive, Transitive but not Symmetric.	
2	Let f: R→R be defined by $f(x) = x^2+1$, then the pre image of 10 is (a) {3} (b) {-3} (c) {3,-3} (d) ∅	
3	A set A has 3 elements and a set B has 4 elements. The number of injective function that can be defined from A to B is (a) 144 (b) 12 (c) 24 (d) 64	
4	Maximum number of equivalence relation on the set A = {1,2,3} are (a)1 (b) 2 (c) 3 (d) 5	
5	The domain of the function f : R → R defined by f (x) = $\sqrt{x^2 - 3x + 2}$ is (a) $(-\infty, 1] \cup [2, \infty)$ (b) $(-\infty, 1]$ (c) $[2, \infty)$ (d) [1,2]	
	ASSERTION-REASON BASED QUESTIONS **In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.** **(a) Both A and R are true and R is the correct explanation of A.** **(b) Both A and R are true but R is not the correct explanation of A.** **(c) A is true but R is false.** **(d) A is false but R is true.**	
1	**Assertion (A)** : f: N→N given by f(x)=5x is injective but not surjective **Reason (R)** : If co-domain ≠ range, then the function is not surjective	
2	**Assertion (A)** : The relation R in the set {1,2,3} given by R = { (1,1),(2,2),(3,3),(1,2),(2,3)} is an equivalence relation **Reason (R) :** A relation R is said to an equivalence relation if R is reflexive, symmetric and transitive.	
3	**Assertion (A)** :Let A = {1, 2, 3,4,5} and B = {a, b}. Then the number of surjections from A into B is 32. **Reason (R) : T**he number of functions that can be defined from A = {1, 2, 3,4,5} to B={a, b} is 32	
4	**Assertion(A) :** $\sin^{-1}(x) = (\sin x)^{-1}$ **Reason (R) :** Any value in the range of principal value branch is called principal value of that inverse trigonometric function.	
5.	**Assertion(A) :** The domain of the function $\sec^{-1}2x$ is $(-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty)$	

	Reason (R) : $\sec^{-1}(-2) = \frac{\pi}{4}$	
	VERY SHORT ANSWER (2MARKS EACH)	
1	Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$	
2	Find the principal value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$.	
3.	Let f: $R \to R$ be defined by f(x) = cosx, show that f is neither one-one nor onto.	
4	Evaluate $\text{Sin}\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$	
5	Evaluate $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right] + \cos^{-1}\left[\cos\left(\frac{13\pi}{7}\right)\right]$	
	SHORT ANSWER (3 MARKS EACH)	
1.	Let f: $N \to N$ be a function defined as $9x^2 + 6x - 5$. Show that f: $N \to S$, where S is the range of f is bijective.	
2.	Let f: $R \to R$ bde the function defined by $f(x) = \frac{1}{2 - \cos x}$ $\forall x \in R$. Then find range of f.	
3.	Evaluate $\text{Cos}\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6}\right)$	
4	Let A = R –{3} , B = R –{1}. If $A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall x \in A. \textit{Show that } f \textit{ is bijective}.$	
5.	Let T be the set of all triangles in a plane The R is defined by R ={(T_1 ,T_2) : T_1 is similar to T_2 : $T_1, T_2 \in T$}. Show that R is an equivalence relation.	
	LONG ANSWER (5 MARKS EACH)	
1	Show that f: $R \to R$ defined by $f(x) = \frac{x}{x^2+1}$ $\forall x \in R$ *is* neither one-one nor onto.	
2	Let R be a relation on N X N defined by (a,b)R(c,d) iff ad(b+c) = bc(a+d). Show that R is an equivalence relation	
3.	Show that the function f in A = R-$\{\frac{2}{3}\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Also if f(x) =y , express x in terms of y.	
4.	Let A= {1,2,3,…,9} and R be the relation in AXA defined by (a,b)R(c,d) if a+d=b+c for (a,b),(c,d) in AxA. Prove that R is an equivalence relation. Also obtain the equivalence class {(2,5)}.	
5	Show that the relation S in the set A= $\{x \in Z : 0 \le x \le 12\}$ *given by* S= $\{(a,b): a,b \in Z, \lvert a-b \rvert \textit{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to2.	
	MATRICES AND DETERMINANTS(ALGEBRA)	
	MCQ (1 MARK EACH)	
1	1. If $\begin{vmatrix} -1 & 2 \\ 4 & 8 \end{vmatrix} = \begin{vmatrix} 2 & x \\ x & -4 \end{vmatrix}$, x = ? (a) $\sqrt{2}$ (b) $\sqrt{4}$ (c) $2\sqrt{2}$(d) $\pm 2\sqrt{2}$	d
2	2. If A is a square matrix such that $A^2 = A$, then the value of $(I+A)^2$	b

	$-3A$ is , where I is an identity matrix (a) $-I$ (b) I (c) A (d) $A-I$	
3	3. If A_{ij} is the cofactor of the elements a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ then the value of $a_{32}.A_{32}$ is (a) -110 (b) 0 (c) -22 (d) 110	a
4	If A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ then |adj(adjA)| (a) = 14^4 (b) 14^3 (c) 14^2 (d) 14	a
5	A is a square matrix of order 3x3 such that |A| =5 then |3A| is equal to (a) 45 (b) 27 (c) 81 (d) 135	d
	VERY SHORT ANSWER (2MARKS EACH)	
1.	If $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix}$ =O find the value of x	
2.	Find a Matrix A if $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} 4 & 8 & 4 \\ 1 & 2 & 1 \\ 3 & 6 & 3 \end{bmatrix}$	
3.	The matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & 1 \\ c & 1 & 0 \end{bmatrix}$ is skew symmetric find a, b and c.	
4	Given A=$\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$ Prove that $A^2 -9A +2I =O$ and express A^{-1} in terms of A	
5	For what value of x the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular.	
	SHORT ANSWER (3 MARKS EACH)	
1	If A=$\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$ Evaluate $A^2 -2A-7I$ and hence calculate A^{-1}	
2	Express $\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices	
3	Find the adjoint of A, A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$	
4	Given A = $\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, B^{-1} = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, compute $(AB)^{-1}$.	
5	Find the equation of the line joining the points (3,1) and (9,3) using determinants	
	LONG ANSWER (5 MARKS EACH)	

1	If A= $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ find A^{-1}, using A^{-1} solve the system of equations 2x-3y+5z =11 ,3x+2y-4z =-5 , x+y-2z = -3.	
2	Using matrices solve the following system of equations $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} + \frac{-20}{z} = 2$	
3	Using matrix method, solve. 8x – 4y + z = 5 , 10x + 6z = 4 , 8x + y + 6z = 5	
4	Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ $\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ and solve the system of equations x+3z =9 , -x +2y -2z= 4 , 2x -3y+4z=-3	
5	The sum of three numbers is 6. If we multiply the third number by 3 and add second number to it , we get 11. By adding the first and third number we get double of second number represent it algebraically and find the numbers using matrix method.	

	ANSWERS	
MCQ	**ANSWERS 1)d 2)c 3)c 4)c 5)a**	
A&R	**ANSWERS 1)a 2)d 3)d 4)d 5)c**	
SA 1.	$\sin^{-1}\left[sin\left(\frac{13\pi}{7}\right)\right] = \sin^{-1}\left[sin\left(2\pi - \frac{\pi}{7}\right)\right]$ $= \sin^{-1}\left[sin\left(-\frac{\pi}{7}\right)\right]$ $= -\frac{\pi}{7}$ ========	
2	$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = \frac{\pi}{3} - (\pi - \sec^{-1}(2)$ $= \frac{\pi}{3} - \pi + \cos^{-1}(\frac{1}{2})$ $= \frac{\pi}{3} - \pi + \frac{\pi}{3}$ $= -\frac{\pi}{3}$ =======	
3	$f(\frac{\pi}{2}) = \cos\frac{\pi}{2}$ $= 0$ $f(-\frac{\pi}{2}) = \cos(-\frac{\pi}{2})$ $= 0$ So f is not one-one $-1 \le cosx \le 1\ \forall\ x \in R$. No pre image for x $\in (-\infty, -1] \cup [1,\infty)$ $\therefore$ f *is not onto.*	

4	$\text{Sin}\left(\frac{\pi}{2} - sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \text{Sin}\left(\frac{\pi}{2} + sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ $= \text{Sin}\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$ $= \sin\left(\frac{5\pi}{6}\right)$ $=\sin\left(\pi - \frac{\pi}{6}\right)$ $= \sin\frac{\pi}{6}$ $= \frac{1}{2}$ =====	
5	$\sin^{-1}\left[sin\left(\frac{13\pi}{7}\right)\right] + \cos^{-1}\left[cos\left(\frac{13\pi}{7}\right)\right]$ $= \sin^{-1}\left[sin\left(2\pi - \frac{\pi}{7}\right)\right] + \cos^{-1}\left[cos\left(2\pi - \frac{\pi}{7}\right)\right]$ $=\sin^{-1}\left[sin\left(-\frac{\pi}{7}\right)\right] + \cos^{-1}\left[cos\left(\frac{\pi}{7}\right)\right]$ $= -\frac{\pi}{7} + \frac{\pi}{7}$ $= 0$ ====	
SA1	One-one $f(x_1) = f(x_2)$ $9x_1^2 +6x_1 -5 = 9x_2^2 +6x_2 -5$ $(x_1 - x_2)(9x_1+9x_2+6) = 0$ $x_1 - x_2 =0$ $x_1 =x_2$ f is one-one. $f(x) = 9x^2 +6x -5$ $y = 9x^2 +6x -5$ $=(3x+1)^2 -6$ $X = \frac{-1 \pm \sqrt{y+6}}{3} \in N \ if \ y \in S.$ f is onto. f is bijective.	
2	$f(x) =\frac{1}{2-cosx}$ $y = \frac{1}{2-cosx}$ we know that,$-1 \leq cosx \leq 1 \ \forall \ x \in R$ $\cos x = 2-\frac{1}{y}$ $-1 \leq 2 - \frac{1}{y} \leq 1$ $-3 \leq -\frac{1}{y} \leq -1$ $\frac{1}{3} \leq y \leq 1$ Range of f is [1/3,1]	
3	$\text{Cos}\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6}\right) = \text{Cos}\left(\pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{\pi}{6}\right)$ $= \text{Cos}\left(\pi - \frac{\pi}{6} - \frac{\pi}{6}\right)$ $=\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ ====	
4	$f(x) =\frac{x-2}{x-3},$ <u>one-one</u>	

	$f(x_1) = f(x_2)$ $\frac{x1-2}{x1-3} = \frac{x2-2}{x2-3}$ $x_1 = x_2$, f is one-one <u>Onto</u> $y = \frac{x-2}{x-3}$ $x = \frac{3y-2}{y-1}$ $f(x) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3}$ f(x) = y f is onto f is bijective.	
5	A relation R in T is said to be an equivalence relation if R is reflexive, symmetric and transitive. Reflexivity Since every triangle is similar to itself ∴ R is reflexive Symmetry $(T_1, T_2) \in \boldsymbol{R}$, T_1 is similar to T_2 T_2 is similar to T_1 , $(T_2, T_1) \in \boldsymbol{R}$ R is symmetric Transitivity T_1 is similar to T_2 , T_2 is similar to T_3 then T_1 is similar to T_3. Hence R is transitive. R is an equivalence relation.	
LA1		

	$f(x_1) = f(x_2)$ $\Rightarrow \dfrac{x_1}{x_1^2+1} = \dfrac{x_2}{x_2^2+1}$ $\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$ $\Rightarrow x_1 x_2 (x_2 - x_1) = x_2 - x_1$ $\Rightarrow x_1 = x_2$ or $x_1 x_2 = 1$ We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$, for instance, i we take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1) = \frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Henc f is not one-one. Also, f is not onto for if so then for $1 \in \mathbf{R}$ $\exists$ $x \in \mathbf{R}$ such that $f(x) =$	
2	A relation R in T is said to be an equivalence relation if R is reflexive, symmetric and transitive. Reflexivity (a,b)∈ $N \times N$ (a,b)R(a,b) if ab(b+a) = ba(a+b) =(a,b)R(a,b) R is reflexive Symmetry (a,b) , $(c, d) \in$ $N \times N$ (a,b)R(c,d) → ad(b+c) = bc(a+d) bc(a+d) = ad(b+c) cb(d+a) = da(c+b) (c,d)R(a,b) Ris symmetric Transitivity (a,b)R(c,d) → ad(b+c) = bc(a+d) Ie, $\frac{1}{b} + \frac{1}{c} = \frac{1}{a} + \frac{1}{d}$ -----(1) (c,d)R(e,f) → cf(d+e) = de(c+f) Ie, $\frac{1}{d} + \frac{1}{e} = \frac{1}{c} + \frac{1}{f}$ ------(2) (1) +(2) → $\frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$ af(b +e) = be (a+f) (a,b)R(e,f) So, R is transitive R is an equivalence relation.	
3	f(x) =$\frac{4x+3}{6x-4}$ one-one f(x) = f(y) $\frac{4x+3}{6x-4} = \frac{4y+3}{6y-4}$ x=y f is one-one onto y = f(x) =$\frac{4x+3}{6x-4}$ x=$\frac{4y+3}{6y-4}$	

	f(x) = $\frac{4\frac{4y+3}{6y-4}+3}{6\frac{4y+3}{6y-4}-4}$ f(x) = y ie, f is onto So fis bijective.	
4	A relation R is said to be an equivalence relation if R is reflexive, symmetric and transitive. (a,b)R(c,d) if a+d=b+c Reflexivity (a,b)∈ $A \times A$ (a,b)R(a,b) if a+b = b+a Ie,(a,b)R(a,b) R is reflexive Symmetry (a,b) , $(c,d) \in A \times A$ (a,b)R(c,d) if a+d=b+c b+c = a+d c+b = d+a (c,d)R(a,b) R is symmetric Transitivity (a,b) , $(c,d),(e,f) \in A \times A$ (a,b)R(c,d) if a+d=b+c -----(1) (c,d)R(e,f) if c+f =d+e ------(2) (1) +(2) → a +f = b +e (a,b) R (e,f) R is transitive. R is an equivalence relation Elements related to (2,5) are {(2,5),(1,4),(3,6),(4,7),(5,8),6,9)}	
5	A relation R is said to be an equivalence relation if R is reflexive, symmetric and transitive Reflexivity |a-a| =0= m(4) $\forall a \in A$, R is reflexive Symmetry (a,b) ∈ $R \implies$ |a-b| = m(4) then |b-a| = m(4) $\implies$ (b,a) ∈ R R is symmetric Transitivity (a,b)&(b,c) ∈ R $\implies$ |a-b| = m(4) and |b-c| = m(4) |a-b+b-c |= m(4) $\pm m(4)$ = m(4) R is transitive Therefore R is an equivalence relation [2] = {2,6,10}	

	UNIT 2	
1 MCQ	**ANSWERS 1)d 2)b 3a 4)a 5)d**	
VSA 1	We have $\begin{bmatrix} 2x & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$ $\Rightarrow \begin{bmatrix} 2x-9 & 4x \end{bmatrix} \begin{bmatrix} x \\ 8 \end{bmatrix} = 0$ $\Rightarrow [2x^2 - 9x + 32x] = [0]$ $\Rightarrow [2x^2 + 23x] = [0]$ $\Rightarrow 2x^2 + 23x = 0$ Or $x(2x+23) = 0$ $\Rightarrow x = 0, x = \frac{-23}{2}$	
2	Let A =[x y z] $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} x\ y\ z \end{bmatrix} = \begin{bmatrix} 4 & 8 & 4 \\ 1 & 2 & 1 \\ 3 & 6 & 3 \end{bmatrix}$ $\begin{bmatrix} 4x & 4y & 4z \\ x & y & z \\ 3x & 3y & 3z \end{bmatrix} = \begin{bmatrix} 4 & 8 & 4 \\ 1 & 2 & 1 \\ 3 & 6 & 3 \end{bmatrix}$ X=1 , y = 2 , z= 1 A = [1 2 1]	

3	Since, A is skew-symmetric matrix. $\therefore A' = -A$ $\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & a & 3 \\ 2 & b & 1 \\ c & 1 & 0 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & 1 \\ -c & -1 & 0 \end{bmatrix}$ By equality of matrices, we get $a = -2, c = -3$ and $b = -b \Rightarrow b = 0$ $a = -2, b = 0$ and $c = -3$	
4	$A^2 - 9A + 2I = O$ (proof) ie, $A - 9I + 2A^{-1} = O$ $2A^{-1} = 9I - A$	
5	For matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ being singular $\begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix} = 0$ $\Rightarrow 20 - 4x - 2x - 2 = 0$ $\Rightarrow 18 - 6x = 0$ $\Rightarrow 6x = 18$ $\Rightarrow x = \frac{18}{6} = 3$	
SA1	$A^2 - 2A - 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $= O$ ie, $A - 2I - 7A^{-1} = O$	

	7 A^{-1} =A-2I $= \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $= \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}$ $A^{-1} = \frac{1}{7} \begin{bmatrix} -1 & 2 \\ 4 & -1 \end{bmatrix}$	
2	$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ A = P+Q $P = \frac{(A + A')}{2}$ $Q = \frac{(A - A')}{2}$ $A' = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$ $P = \frac{(A + A')}{2} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ $Q = \frac{(A - A')}{2} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$ $P' = P \text{ and } Q' = Q$	
3	$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ $\text{AdjA} = \begin{bmatrix} 6 & -2 & -3 \\ 1 & -5 & 3 \\ -5 & 4 & -1 \end{bmatrix}$	
4	$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ $\|A\|=-3$ $A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ $(AB)^{-1} = B^{-1} A^{-1}$ $= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ $= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$	

5	Let $p(x, y)$ be any point on the line joining $(3, 1)$ and $(9, 3)$ $\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$ $R_2 \Rightarrow R_2 - R_1; R_3 \Rightarrow R_3 - R_1$ $\begin{vmatrix} x & y & 1 \\ 3-x & 1-y & 0 \\ 9-x & 3-y & 0 \end{vmatrix} = 0$ $\Rightarrow 1[(3-x)(3-y)-(1-y)(9-x)] = 0$ $\Rightarrow 9-3y-3x+xy+x-9+9y-xy = 0$ $\Rightarrow 6y-2x = 0$ $\Rightarrow x = 3y$	
LA 1	$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix},$ $\lvert A\rvert = 2(-4+4)+3(-6+4)+5(3-2)$ $= -6+5$ $= -1$ $\text{Adj}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $\therefore A^{-1} = \frac{\text{Adj}A}{\lvert A\rvert} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$	

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

$$\therefore \text{Let } A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\therefore AX = B$$

$$\Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore x = 1, y = 2, z = 3$$

2	Let $\frac{1}{x} = p;\ \frac{1}{y} = q;\ \frac{1}{z} = r$ $2p + 3q + 10r = 4$ $4p - 6q + 5r = 1$ $6p + 9q - 20r = 2$ $AX = B \Rightarrow X = A^{-1}B$ $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, \lvert A \rvert = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$ $= 2(120 - 45) - 3(-60 - 30) + 10(36 + 36) = 1200$ $\text{co factor of } A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}$ $A^{-1} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 0 \\ 72 & 0 & -24 \end{bmatrix}$ $X = A^{-1}B = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 0 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ $\Rightarrow x = 2, y = 3, z = 5$	
3	AX=B $\begin{bmatrix} 8 & -4 & 1 \\ 10 & 0 & 6 \\ 8 & 1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 5 \end{bmatrix}$	

	$A^{-1} = \frac{1}{10}\begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}$ $X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -6 & 25 & -24 \\ -12 & 40 & -38 \\ 10 & -40 & 40 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \\ 5 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -30+100-120 \\ -60+160-190 \\ 50-160+200 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} -50 \\ -90 \\ 90 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -9 \\ 9 \end{bmatrix}$	
4	C = $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ CD = I D= C^{-1} x+3z =9 , -x +2y -2z= 4 , 2x -3y+4z=-3 AX= B A=$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}$ B=$\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$ A = C^T A^{-1} $=(C^T)^{-1}$ $=(C^{-1})^T$ $=(D)^T$ $=\begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}$ X = A^{-1} B $=\begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$ $=\begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$	

5

Let the first , second & third number be x, y, z respectively

Given,

$\therefore\ x + y + z = 6$

$y + 3z = 11$

$x + z = 2y$ or $x - 2y + z = 0$

Hence $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ & $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Hence, $\operatorname{adj}(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$

Now ,

$$A^{-1} = \frac{1}{|A|}\operatorname{adj}(A)$$

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Solution of given system of equations is

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42 & -33 & +0 \\ 18 & +0 & +0 \\ -6 & +33 & +0 \end{bmatrix}$$

	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\therefore\ x = 1, y = 2, z = 3$	

Chapter Continuity and Differentiability

MCQs (1M)

1) **If $y = sin^{-1}x$, then $(1 - x^2)y_2$ is equals to,**
   a) $\boldsymbol{xy_1}$ **(b)** $\boldsymbol{xy}$ **(c)** $\boldsymbol{xy_2}$ **(d)** $\boldsymbol{x^2}$

2) The value of k for which the function, $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at $x = 0$,

   (a) 0 (b) -1 **(c) 1** (d) 2

3) Which of the following is not a continuous function in their domain,
   (a) $\sin x$ (b) $|x + 2|$ (c) e^x (d)$[x]$

4) If $x = at^2, y = 2at$, then $\frac{dy}{dx}$ is equal to
   a) t b) 1/t c) $-1/t^2$ d) None

5) The derivative of $\tan^{-1}\left(\frac{sinx}{1+cosx}\right)$ with respect to x is

   a)0 b) 1 c) -1 d) 2

VSQ(2M)

6) Find derivative of $\sin(\log x)$ $w.r.t.$ $\log(x^2)$.

7) Find the derivative of

$\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w.r.to x

8)

Find derivative of $f(e^{\tan x})$ w.r.t. x, at $x = 0$, given that, $f'(1) = 5$.

9) If $y = \log(x + \sqrt{1+x^2})$ prove that $y_2 = \frac{-2x}{x^2+1}$

10) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

SA(3M)

11) If y = $x^{\sin x} + (\sin x)^x$ find $\frac{dy}{dx}$

12) If y = $x^{x^{x\cdots\cdots\infty}}$, find $\frac{dy}{dx}$

13) If $y = e^{a\cos^{-1}x}$ Prove that, $(1-x^2)\,y_2 - x\,y_1 - a^2 y = 0$

14)If the function f(x) given by

$$f(x) = \begin{cases} 3ax+b, & \text{if } x>1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$$

is continuous at x = 1, then find the values of a and b

15) If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$ show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

16) If y = $\left(x + \sqrt{1+x^2}\right)^n$, then show that $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y$.

17) If y = $\log[x + \sqrt{x^2 + a^2}]$, then show that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$ (

18) If $y = (\tan^{-1} x)^2$, then show that $(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} = 2$ (

19) If $x = a \cos \theta + b \sin \theta$, $y = a \sin \theta - b \cos \theta$, then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.
.

20) If $x = a \sec^3 \theta$ and $y = a \tan^3 \theta$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

==

Continuity and Differentiability

(Complete solutions)

MCQ (1 M)

1) a

2) c

3) d

4) b

5) a

VSA (2 M) 6)

:Let $A = \sin(\log x)$

$$\rightarrow \frac{dA}{dx} = \frac{\cos(\log x)}{x}$$

Let $B = \log(x^2) = 2\log x$

$$\rightarrow \frac{dB}{dx} = \frac{2}{x}$$

Hence, $\frac{d[\sin(\log x)]}{d[\log(x^2)]} = \frac{dA}{dB} = \frac{\cos(\log x)}{2}$

7)

$let\ y = sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$$\rightarrow y = sin^{-1}\left(\frac{2.\ 2^x}{1+(2^x)^2}\right)$$

Let, $2^x = \tan\theta$

$$\rightarrow 2^x \log 2 = sec^2\theta \frac{d\theta}{dx}$$

$$\rightarrow \frac{d\theta}{dx} = \frac{2^x \log 2}{sec^2\theta}$$

Hence, $y = sin^{-1}\left(\frac{2.\tan\theta}{1+\tan^2\theta}\right) = sin^{-1}(\sin 2\theta) = 2\theta$

$$\rightarrow \frac{dy}{dx} = 2\frac{d\theta}{dx} = 2\frac{2^x \log 2}{sec^2\theta} = \frac{2^{x+1}\log 2}{1+tan^2\theta} = \frac{2^{x+1}\log 2}{1+4^x}$$

8)

Let, $A = f(e^{\tan x})$

$$\rightarrow \frac{dA}{dx} = f'(e^{\tan x}).\,e^{\tan x}.\,sec^2x$$

At $x = 0$, $\frac{dA}{dx} = f'(e^{\tan 0}).\,e^{\tan 0}.\,sec^2 0$

$$\rightarrow \frac{dA}{dx} = f'(1).(1).(1) = 5$$

9)

$y_1 = \frac{1}{x+\sqrt{x^2+1}}\left[1+\frac{2x}{2\sqrt{x^2+1}}\right] = \frac{1}{\sqrt{x^2+1}}$

$$\rightarrow y_2 = \frac{-2x}{x^2+1}$$

10)

Since, $x\sqrt{1+y}+y\sqrt{1+x}=0$

$$\to x\sqrt{1+y}=-y\sqrt{1+x}$$
$$\to x^2(1+y)=y^2(1+x)$$
$$\to x^2+x^2y=y^2+y^2x$$
$$\to x^2-y^2+x^2y-y^2x=0$$

$$\to (x+y)(x-y)+xy(x-y)=0$$
$$\to (x-y)(x+y+xy)=0$$

(i) $x-y=0$ it is not possible

or (ii) $x+y+xy=0$

$$\to y(1+x)=-x$$
$$\to y=\frac{-x}{1+x}$$
$$\to \frac{dy}{dx}=\frac{(1+x)(-1)-(-x)(1)}{(1+x)^2}=\frac{-1}{(1+x)^2}$$

11)

Let, $A=x^{\sin x}$ and $B=(\sin x)^x$

So, $y=A+B$

$$\to \frac{dy}{dx}=\frac{dA}{dx}+\frac{dB}{dx}$$

Since, $A=x^{\sin x}$

$$\log A=\sin x\,.\log x$$
$$\to \frac{1}{A}\frac{dA}{dx}=\cos x\,.\log x+\frac{\sin x}{x}$$
$$\to \frac{dA}{dx}=x^{\sin x}\left(\cos x\,.\log x+\frac{\sin x}{x}\right)$$

Since, $B=(\sin x)^x$

$$\to \log B=x\,.\log(\sin x)$$
$$\to \frac{1}{B}\frac{dB}{dx}=x\,.\cot x+\log(\sin x)$$
$$\to \frac{dB}{dx}=(\sin x)^x(x\,.\cot x+\log(\sin x))$$

Hence, $\frac{dy}{dx}=x^{\sin x}\left(\cos x\,.\log x+\frac{\sin x}{x}\right)+(\sin x)^x(x\,.\cot x+\log(\sin x))$

12)

$y = x^{x^{x^{\cdots\infty}}}$

$$\rightarrow y = x^y$$

$$\rightarrow \log y = y \log x$$

$$\rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{y}{x} + \log x \frac{dy}{dx}$$

$$\rightarrow \frac{dy}{dx}\left[\frac{1}{y} + \log x\right] = \frac{y}{x}$$

$$\rightarrow \frac{dy}{dx}\left[\frac{1 + y\log x}{y}\right] = \frac{y}{x}$$

$$\rightarrow \frac{dy}{dx} = \frac{y^2}{x(1 + y\log x)}$$

13) **y** $= e^{a\cos^{-1}x}$

$$y_1 = e^{a\cos^{-1}x} \times \frac{-a}{\sqrt{1-x^2}}$$

$$\rightarrow \sqrt{1-x^2}\, y_1 = -ay$$

$$\rightarrow \sqrt{1-x^2}\, y_2 - \frac{2xy_1}{2\sqrt{1-x^2}} = -ay_1$$

$$\rightarrow \sqrt{1-x^2}\, y_2 - \frac{2xy_1}{2\sqrt{1-x^2}} = \frac{a^2y}{\sqrt{1-x^2}}$$

$$\rightarrow (1-x^2)y_2 - xy_1 = a^2y$$

14)

Given, $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$

is continuous at $x = 1$.

$\therefore \quad \text{LHL} = \text{RHL} = f(1) \quad \ldots(i)$

Now, $\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (5ax - 2b)$

$= \lim_{h \to 0} [5a(1 - h) - 2b]$

[put $x = 1 - h$; when $x \to 1^-$, then $h \to 0$]

$= \lim_{h \to 0} (5a - 5ah - 2b) = 5a - 2b$

and $\text{RHL} = \lim_{x \to 1^+} (3ax + b) = \lim_{h \to 0} [3a(1 + h) + b]$

[put $x = 1 + h$; when $x \to 1^+$, then $h \to 0$]

$= \lim_{h \to 0} (3a + 3ah + b) = 3a + b$

Also, given that $f(1) = 11$

On substituting these values in Eq. (i), we get
5a – 2b = 3a + b = 11
⇒ 3a + b = 11 …… (ii)

and 5a – 2b = 11 ……. (iii)
On subtracting 3 × Eq. (iii) from 5 × Eq. (ii), we get
15a + 5b – 15a + 6b = 55 – 33
⇒ 11b = 22 ⇒ b = 2
On putting the value of b in Eq. (ii). we get
3a + 2 = 11 ⇒ 3a = 9 = a = 3
Hence, a = 3 and b = 2

15)

$\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$

on differentiating both sides w.r.t. x, we get

$$\frac{1}{x^2+y^2}\left(2x + 2y\frac{dy}{dx}\right) = 2\cdot\frac{1}{1+\frac{y^2}{x^2}}\left(\frac{y'\cdot x - y}{x^2}\right)$$

$$\Rightarrow \quad \frac{2(x + y\cdot y')}{x^2+y^2} = \frac{2x^2}{x^2+y^2}\left(\frac{y'x - y}{x^2}\right)$$

$$\left[\because y' = \frac{dy}{dx}\right]$$

$$\Rightarrow \quad x + y\cdot y' = y'x - y$$

$$\Rightarrow \quad y'(x - y) = x + y$$

$$\Rightarrow \quad y' = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x-y}$$

Hence proved.

16)

Given, $y = \left(x + \sqrt{1+x^2}\right)^n$ ...(i)

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)$$

[by using chain rule of derivative]

$$\Rightarrow \quad \frac{dy}{dx} = n(x + \sqrt{1+x^2})^{n-1}\left(\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{n(x+\sqrt{1+x^2})^n}{\sqrt{1+x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{ny}{\sqrt{1+x^2}} \quad \text{[from Eq. (i)]}$$

$$\Rightarrow \quad \sqrt{1+x^2}\,\frac{dy}{dx} = ny \qquad \ldots(\text{ii})$$

Again, differentiating both sides w.r.t. x, we get

$$\sqrt{1+x^2}\,\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{1+x^2}} \cdot \frac{dy}{dx} = n\,\frac{dy}{dx}$$

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = n \cdot \sqrt{1+x^2}\,\frac{dy}{dx}$$

[multiplying both sides by $\sqrt{1+x^2}$]

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = n\sqrt{1+x^2} \cdot \frac{ny}{\sqrt{1+x^2}}$$

[from Eq. (ii)]

$$\therefore \ (1+x^2)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = n^2y$$

Hence proved.

17)

Given, $\quad y = \log[x+\sqrt{x^2+a^2}]$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\,\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$\left[\because \frac{d}{dx}(\log f(x)) = \frac{1}{f(x)}\frac{d}{dx}f(x)\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{2x}{2\sqrt{x^2+a^2}}\right)$$

$$\left[\because \frac{d}{dx}(\sqrt{x^2+a^2}) = \frac{1}{2\sqrt{x^2+a^2}} \times 2x\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+a^2}}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}(\sqrt{x^2+a^2}) = 1$$

Again, on differentiating both sides w.r.t. x, we get

$$\sqrt{x^2+a^2}\,\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} \cdot \frac{d}{dx}(\sqrt{x^2+a^2}) = \frac{d(1)}{dx}$$

[by using product rule of derivative]

$$\Rightarrow \quad \frac{d^2y}{dx^2}(\sqrt{x^2+a^2}) + \frac{dy}{dx}\,\frac{1 \cdot 2x}{2\sqrt{x^2+a^2}} = 0$$

On multiplying both sides by $\sqrt{x^2+a^2}$, we get

$$\frac{d^2y}{dx^2}(\sqrt{x^2+a^2})^2 + \frac{dy}{dx} \times \frac{x\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}} = 0$$

$$\therefore \quad (x^2+a^2)\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = 0$$

Hence proved.

18)

Given, $y = (\tan^{-1} x)^2$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = 2\tan^{-1} x . \frac{1}{1+x^2} \quad \left[\because \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\tan^{-1} x}{1+x^2} \Rightarrow (1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$$

Again, differentiating both sides w.r.t. x, we get

$$(1+x^2)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx}\cdot\frac{d}{dx}(1+x^2) = \frac{d}{dx}(2\tan^{-1} x)$$

[by using product rule of derivative]

$$\Rightarrow \quad (1+x^2).\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 2x = \frac{2}{1+x^2}$$

$$\left[\because \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}\right]$$

On multiplying both sides by $(1+x^2)$, we get

$$(1+x^2)^2\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 2x\cdot(1+x^2) = \frac{2}{1+x^2}\cdot(1+x^2)$$

$$\Rightarrow \quad (1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$$

19)

Given x = a cos θ + b sin θ, ... (i)

and y = a sin θ − b cos θ .... (ii)

On differentiating both sides of Eqs. (i) and (ii) w.r.t. θ, we get

$$\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta \qquad \text{...(iii)}$$

and $$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta \qquad \text{...(iv)}$$

Now, $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta + b\sin\theta}{b\cos\theta - a\sin\theta}$$

[dividing Eq. (iv) by Eq. (iii)]

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{-y} \text{ [from Eqs. (i) and (ii)] ...(v) } \mathbf{(1)}$$

Again, differentiating both sides of Eq. (v) w.r.t. x, we get

$$\frac{d^2y}{dx^2} = -\left[\frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2}\right]$$

[by using quotient rule of derivative]

$$\Rightarrow \quad y^2 \frac{d^2y}{dx^2} = -\left[y - x\frac{dy}{dx}\right]$$

$$\therefore \qquad y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

20)

Given, $y = a\tan^3\theta$ and $x = a\sec^3\theta$

On differentiating w.r.t. θ, we get

$$\frac{dy}{d\theta} = 3a\tan^2\theta \frac{d}{d\theta}(\tan\theta)$$

and $$\frac{dx}{d\theta} = 3a\sec^2\theta \frac{d}{d\theta}(\sec\theta)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$$

and $\frac{dx}{d\theta} = 3a\sec^2\theta\sec\theta\tan\theta$

$$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\tan^2\theta\sec^2\theta}{3a\sec^2\theta\sec\theta\tan\theta}$$

$$\frac{dy}{dx} = \frac{\tan\theta}{\sec\theta} = \sin\theta$$

Again differentiating both sides w.r.t. x, we get

$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d}{dx}(\sin\theta) = \frac{d}{d\theta}(\sin\theta)\frac{d\theta}{dx}$$

$$= \frac{\cos\theta}{3a\sec^3\theta\tan\theta} = \frac{\cos^5\theta}{3a\sin\theta}$$

At $\theta = \frac{\pi}{3}$,

$$\frac{d^2y}{dx^2} = \frac{\cos^5\frac{\pi}{3}}{3a\sin\frac{\pi}{3}} = \frac{\left(\frac{1}{2}\right)^5}{3a\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{1\times 2}{2^5\times 3a\sqrt{3}} = \frac{1}{48\sqrt{3}a}$$

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Application of derivatives

MCQ (1M)

1) The real function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) Strictly increasing in $(-\infty, -2)$ and strictly decreasing in $(-2, \infty)$

(b) Strictly decreasing in $(-2, 3)$

© Strictly decreasing in $(-\infty, 3)$ and strictly increasing in $(3, \infty)$

(d) Strictly decreasing in $(-\infty, -2) \cup (3, \infty)$

2) If $y = x(x - 3)^2$ decreases for the values of 'x' given by

(a) $1 < x < 3$ (b) $x < 0$ (c) $x > 0$ (d) $0 < x < 3/2$

3) The function $f(x) = \tan x - x$

(a) Always increases (b) Always decreases (c) Never increases (d) Sometimes increases and sometimes decreases

4) The function $f(x) = \log x$ is strictly increasing on

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, \infty)$ (d) None

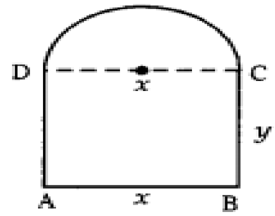
5)The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has

(a) Two points of local maximum (b) Two points of local minimum (c) One maxima and one minima (d) No maxima or minima

VSA (2M)

6) **Side of an equilateral triangle expands at the rate of 2 cm/sec. Find the rate of increase of its area when each side is 10 cm .**
7) Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.
8) The radius r of the base of a right circular cone is decreasing at the rate of 2 cm/min and its height h is increasing at the rate of 3 cm/min. When r = 3.5 cm and h = 6 cm, find the rate of change of the volume of the cone. [Use $\pi = 22/7$]
9) Find the maximum and minimum values of the function given by $f(x) = -(x - 1)^2 + 10$.

10) A window is in the form of a rectangle surmounted by a semi-circular opening . If the perimeter of the window is 10 m , Express y in terms of x



SA (3M)

11) Find the intervals in which the function f(x) = $-2x^3 - 9x^2 - 12x + 1$ is
(i) strictly increasing
(ii) strictly decreasing,

12) Find the intervals in which the function given by f(x) = $x^4 - 8x^3 + 22x^2 - 24x + 21$ is
(i) increasing,
(ii) decreasing.

13) Find the intervals in which the function given by f(x) = sin x + cos x, $0 \le x \le 2\pi$ is
(i) increasing
(ii)decreasing.

14) A ladder 5 m long is leaning against a wall. Bottom of ladder is pulled along the ground away from wall at the rate of 2 m/s. How fast is the height on the wall decreasing, when the foot of ladder is 4 m away from the wall?

15) Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point (1, 4).

CBQ (4M)

16) Sonam wants to prepare a sweet box for Diwali at home. For making lower part of box, she takes a square piece of card board of side 18cm. based on the above information, answer the following questions.

Based on the above information, Answer the following questions.

1. If x cm be the length of each side of the square cardboard which is to be cut off from corner of the square piece of side 18 cm, then x must lie in

A [0, 18] B. (0, 9) C. (0, 3) D. None of these.

2 . Volume of the open box formed by folding up the cutting corner can be expressed as

A. $V = x(18 - 2x)(18 - 2x)$

B. $V = x/2(18 + x)(18 - x)$
C. $V = x/3(18 - 2x)(18 + 2x)$
D. $V = x(18 - 2x)(18 - x)$

3 The values of x for which $\frac{dV}{dx}=0$ are

A. 3, 2 B. 0, 3 C. 0, 9 D. 3, 9

4 Sonam is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum

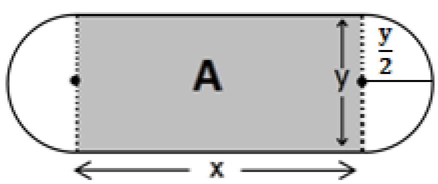
A. 13 cm B. 8 cm C. 3 cm D. 2 cm

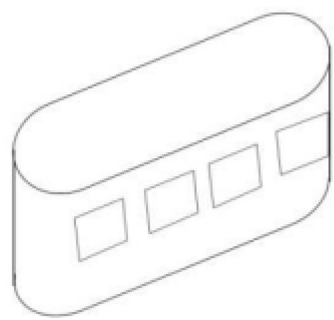
5. The maximum value of the volume is

A. 144 cm^3 B. 232 cm^3 C. 256 cm^3 D. 432 cm^3

17. An architect designs a building for a multi-national company. The floor consists of a rectangular region with semicircular ends having a perimeter of 200m as shown below:

Design of Floor Building





Based on the above information answer the following:

1 If x and y represent the length and breadth of the rectangular region, then the relation between the variables is

A. x + π y = 100 B. 2x + π y = 200 C. π x + y = 50 D.x + y = 100

2. The area of the rectangular region A expressed as a function of x is

A. $\frac{2}{\pi}(100x-x^2)$ B. $\frac{1}{\pi}(100x-x^2)$ C. $\frac{x}{\pi}(100-x)$ D. $\pi y^2 + \frac{2}{\pi}(100x-x^2)$

3. The maximum value of area A is

A. $\frac{\pi}{3200}$ m^2

B $\frac{3200}{\pi}$ m^2

C $\frac{5000}{\pi}$ m^2

D $\frac{1000}{\pi}$ m^2

18. A company makes closed water storage tank. The water tank is cylindrical in shape. Let

S be the given surface area ,r be the radius of base and h be height of the tank . Based on the information provided, answer the following :

1. Relation between S ,r and h is:

   (a) $S = 2\pi rh + 2\pi r^2$

   (b) $S = 2\pi rh + \pi r^2$

   (c) $S = \pi r^2 h + \pi r^2$

   (d) $S = \pi r^2 h + 2\pi r^2$

2. Volume of the tank in terms of S and r is :

$\frac{rs-\pi r^3}{2}$ — $\frac{rs-\pi r^2}{2}$ $\frac{rs-2\pi r^3}{2}$ $\frac{rs-\pi r^3}{3}$

-

3. dV/dr

   (a) $\frac{S+3\pi r^2}{2}$

   (b) $\frac{S-6\pi r^2}{2}$

   (c) $\frac{S+2\pi r^2}{3}$

   (d) $\frac{S-2\pi r^2}{3}$

4. If the volume is maximum ,then the relation between r and h is:
   (a) h=r
   (b) h=3r
   (c) r=3h
   (d) h=2r

Solutions : Application of derivatives

1) c

2) a

3) a

4) a

5) c

SA(2M)

6) $\frac{dx}{dt}$ = 2 cm/sec A= $\frac{\sqrt{3}}{4}x^2$ $\frac{dA}{dt} = \frac{\sqrt{3}}{4} . 2x \frac{dx}{dt}$ = 10 $\sqrt{3}$ cm^2/sec

7) Given, $f(x) = x^3 - 3x^2 + 6x - 100$
On differentiating both sides w.r.t. x, we get
$f'(x) = 3x^2 - 6x + 6 = 3x^2 - 6x + 3 + 3$
$= 3(x^2 - 2x + 1) + 3 = 3(x - 1)^2 + 3 > 0$
$\therefore f'(x) > 0$ This shows that the function is increasing on R

8)

Given $\frac{dr}{dt} = -2$ cm/min (decreasing)

and $\frac{dh}{dt} = 3$cm/min (increasing)...(*i*)

Volume of cone $V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi \frac{d}{dt}(r^2 h)$$

$$= \frac{1}{3}\pi\left[r^2 \cdot \frac{dh}{dt} + 2rh\frac{dr}{dt}\right]$$

$$= \frac{1}{3}\pi(3r^2 - 4rh) \text{ [using } (i)]$$

$$\therefore \left.\frac{dV}{dt}\right]_{\substack{r=3.5 \text{ cm} \\ h=6 \text{ cm}}} = \frac{1}{3}\pi$$

$$[3(3.5)^2 - 4 \times 3.5 \times 6]$$

$$= \frac{1}{3}\pi(3 \times 12.25 - 84)$$

$$= \frac{1}{3}\pi(36.75 - 84)$$

$$= \frac{1}{3}\pi \times (-47.25)$$

.

$$= -15.75\ \pi \text{ cm}^3/\text{min}$$

Volume is decreasing at the rate of 15.75π cm^3/min.

9) $-(x - 1)^2 < 0$ for $x \in R$
$\Rightarrow -(x - 1)^2 + 10 \leq 10$
$\Rightarrow f(x) \leq 10$,
Maximum value = 10.
Minimum value = Nil.

10) Since, perimeter of the window is 10 m.
$\therefore 2x + y + y + \pi x/2 = 10$

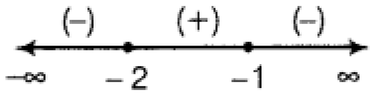
$y = 5 - x - \pi x/4$

SA(3M)

11)

Given, $f(x) = -2x^3 – 9x^2 – 12x + 1$
On differentiating both sides w.r.t. x, we get
$f'(x) = – 6x^2 – 18x – 12$
$\Rightarrow f'(x) = – 6(x^2 + 3x + 2)$
$\Rightarrow f'(x) = – 6(x^2 + 2x + x + 2)$
$\Rightarrow – 6 [x(x + 2) + 1 (x + 2)]$
$\Rightarrow – 6 (x + 2) (x + 1)$

Now, put $f'(x) = 0$
$\Rightarrow -6(x + 2)(x + 1) = 0$
$\Rightarrow x = – 2, -1$
The points, $x = – 2$ and $x = – 1$ divide the real line into their disjoint intervals $(-\infty,-2)$, $(- 2, -1)$ and $(- 1, \infty)$,



The nature of function in these intervals are given below:

Interval	Sign of f'(x) F'(x) = -6(x + 2)(x + 1)	Nature of function
$(-\infty, -2)$	(-) (-) (-) = (-) < 0	Strictly decreasing
$(-2, -1)$	(-) (+) (-) = (+) > 0	Strictly increasing
$(-1, \infty)$	(-) (+) (+) = (-) < 0	Strictly decreasing

Hence, f(x) is strictly increasing in the interval $(- 2, -1)$ and f(x) is strictly decreasing in the interval $(- \infty, – 2) \cup (-1, \infty)$.

12)

Given function is
$f(x) = x^4 – 8x^3 + 22x^2 – 24x + 21$
On differentiating both sides w.r.t. x, we get
$f(x) = 4x^3 – 24x^2 + 44x – 24$
$= 4(x^3 – 6x^2 + 11 x – 6)$
$= 4(x – 1) (x^2 – 5x + 6)$
$= 4(x – 1) (x – 2) (x – 3)$
Put $f'(x) = 0$
$\Rightarrow 4(x – 1)(x – 2)(x – 3) = 0$
$\Rightarrow x = 1, 2, 3$
So, the possible intervals are $(-\infty, 1)$, $(1, 2)$, $(2, 3)$ and $(3, \infty)$.
For interval $(- \infty, 1)$, $f'(x) < 0$ For interval $(1, 2)$, $f'(x) > 0$
For interval $(2, 3)$, $f'(x) < 0$ For interval $(3, \infty)$, $f'(x) > 0$.

Also, as f(x) is a polynomial function, so it is continuous at x = 1, 2, 3…………
Hence,
(i) function increases in $[1, 2]$ and $[3, \infty)$.
(ii) function decreases in $(-\infty, 1]$ and $[2, 3]$

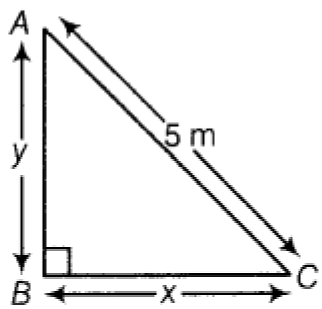
13)

Given function is $f(x) = \sin x + \cos x$.
On differentiating both sides w.r.t. x, we get
$f'(x) = \cos x – \sin x$
Now, put $f'(x) = 0 \Rightarrow \cos x – \sin x = 0$
$\Rightarrow \tan x = 1 \Rightarrow x = \pi/4, 5\pi/4$, as $0 \le x \le 2\pi$
Now, we find the intervals in which f(x) is strictly increasing or strictly decreasing

Interval	Test value	$f'(x) = \cos x - \sin x$	Sign of $f'(x)$
$0 < x < \frac{\pi}{4}$	At $x = \frac{\pi}{6}$	$\cos\frac{\pi}{6} - \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$	+ve
$\frac{\pi}{4} < x < \frac{5\pi}{4}$	At $x = \frac{\pi}{2}$	$\cos\frac{\pi}{2} - \sin\frac{\pi}{2} = 0 - 1 = -1$	–ve
$\frac{5\pi}{4} < x < 2\pi$	At $x = \frac{3\pi}{2}$	$\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2} = 0 - (-1) = 1$	+ve

Note that, $f'(x) > 0$ in $(0, \pi/4)$, $f'(x) < 0$ in, $(\pi/4, 5\pi/4)$ and $f'(x) > 0$ in $(5\pi/4, 2\pi)$.
Since, $f(x)$ is a trigonometric function, so it is
continuous at $x = 0, \pi/4, 5\pi/4$ and 2π.
Hence, the function is
(i) increasing in $[0, \pi/4]$ and $[5\pi/4, 2\pi]$
(ii) decreasing in $[\pi/4, 5\pi/4]$

14)
First, draw a rough figure of a right angled triangle, then use Pythagoras theorem.
Further differentiate the relation between sides with respect tof and ; simplify it
Let AC be the ladder, BC = x and height of the wall, AB = y.



As the ladder is pulled along the ground away
from the wall at the rate of 2 m/s.
So, dydx = 2m/s
To find ,when x = 4.
In right angled MBC, by Pythagoras theorem, we get
$(4)^2 + (BC)^2 = (AC)^2$
$x^2 + y^2 = 25$
$(4)^2 + y^2 = 25$
$16 + y^2 = 25$
$y^2 = 9$
$y = \sqrt{9}$ [taking positive square root]
$\therefore y = 3$
On differentiating both sides of Eq. (i) w.r.t. t, we get

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \qquad \text{...(ii)}$$

[dividing both sides by 2]

On substituting the values of x, y and $\frac{dx}{dt}$

in Eq. (ii), we get

$$(4 \times 2) + 3 \times \frac{dy}{dt} = 0$$

$$\Rightarrow \qquad 8 + 3 \times \frac{dy}{dt} = 0$$

$$\therefore \qquad \frac{dy}{dt} = \frac{-8}{3} \text{ m/s}$$

Hence, height of the wall is decreasing at the rate of 83m/s.

Note: In a rate of change of a quantity, +ve sign shows that it is increasing and – ve sign shows that it is decreasing.

15)

The given equation of curve is $y^2 = 2x$ and the given point is Q (1,4).
Let P(x, y) be any point on the curve.
Now, distance between points P and Qis given by

$$PQ = \sqrt{(1-x)^2 + (4-y)^2}$$

$$\left[\begin{array}{l}\text{using distance formula,}\\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\end{array}\right]$$

$$\Rightarrow \quad PQ = \sqrt{1 + x^2 - 2x + 16 + y^2 - 8y}$$

$$= \sqrt{x^2 + y^2 - 2x - 8y + 17}$$

On squaring both sides, we get

$$PQ^2 = x^2 + y^2 - 2x - 8y + 17$$

$$\Rightarrow \quad PQ^2 = \left(\frac{y^2}{2}\right)^2 + y^2 - 2\left(\frac{y^2}{2}\right) - 8y + 17$$

$$\left[\text{given, } y^2 = 2x \Rightarrow x = \frac{y^2}{2}\right]$$

$$\therefore \quad PQ^2 = \frac{y^4}{4} + y^2 - y^2 - 8y + 17$$

$$\Rightarrow \quad PQ^2 = \frac{y^4}{4} - 8y + 17$$

Let $PQ^2 = Z$

Then, $Z = y^4/4 - 8y + 17$

On differentiating both sides w.r.t. y, we get

$dZ/dy = 4y^3/4 - 8 = y^3 - 8$

For maxima or minima, put $dZ/dy = 0$

$\Rightarrow y^3 - 8 = 0 \Rightarrow y^3 = 8$

$\Rightarrow y = 2$

Also, $d^2Z/dy = 3y^2$

On putting y = 2, we get

$(d^2Z/dy^2)_{y=2} = 3(2)^2 = 12 > 0$

$d^2Z/dy^2 > 0$

$\therefore$ Z is minimum and therefore PQ is also minimum as $Z = PQ^2$.

On putting y = 2 in the given equation, i.e. $y^2 = 2x$, we get

$(2)^2 = 2x$

$\Rightarrow 4 = 2x$

$\Rightarrow x = 2$

Hence, the point which is at a minimum distance from point (1, 4) is P (2, 2).

CBQ (4M)

16) 1.B 2.A 3.D 4.C 5,D

17) 1. B 2. A 3. C 4. A 5.D

18) 1. A 2. C 3 B 4. D

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Chapter 7

Integration

Multiple choice questions

1. If $\frac{d}{dx}f(x) = \frac{1}{\sqrt{x}}$ and f(1)= 2 then f(x) =
   a) $\frac{1}{2\sqrt{x}}$ b) $2\sqrt{x}$ c) $2\sqrt{x}+2$ d) $\frac{1}{2\sqrt{x}}+2$
2. $\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{\sqrt{1-x^2}}dx$
   a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$
3. If $\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{8}$, find 'a'.
   a) 1 b) 2 c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$
4. If f(x) = $\int_0^x xsinxdx$, then find $\frac{d}{dx}f(x)$=
   a) x sinx b) xcosx+sinx c) –xcosx+sinx d) None of these
5. If $\int \frac{x-1}{x^2}e^x dx = e^x f(x) + c$ then f(x)=
   a) $\frac{1}{x^2}$ b) $\frac{1}{x}$ c) $-\frac{1}{x^2}$ d) $-\frac{1}{x}$

2 Marks questions

1. Evaluate: $\int \frac{dx}{x+xlogx}$ or $\int \frac{sinx}{\sin(x+a)}$ dx
2. Evaluate: $\int \frac{dx}{2x^2-3x+7}\ dx$
3. Evaluate: $\int \frac{dx}{\sqrt{7-6x-x^2}}$
4. Evaluate: $\int [\frac{1}{logx} - \frac{1}{(logx)^2}]dx$ or $\int \frac{logx}{(1+logx)^2}dx$
5. Evalaute: $\int_{-5}^{5}|x+2|$dx or $\int_{-2}^{1}|x^3-x|$dx
6. Evalaute: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{tanx}}$ dx or $\int_3^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{8-x}}$ dx
7. Evalaute: $\int_0^1 \log(\frac{1-x}{x})dx$ or $\int_0^2 x\sqrt{2-x}\,dx$
8. Evalaute: $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}$ dx or $\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}}dx$

3 marks questions

1. Evaluate: $\int \frac{5x-2}{1+2x+3x^2}$ dx

2. $\int \frac{3x+5}{\sqrt{x^2-8x+7}} dx$
3. Evaluate: $\int \frac{1-x^2}{x(1-2x)} dx$
4. Evaluate: $\int \frac{dx}{x(x^7+5)}$ or $\int \frac{dx}{e^x-1}$
5. Evaluate: $\int \frac{2x-1}{(x-1)(x-2)(x-3)} dx$ or $\int \frac{2x-1}{(x-1)^2(x-2)} dx$
6. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+4)} dx$ or $\int \frac{1}{(x^2+1)(x^2+4)} dx$
7. Evaluate: $\int \frac{2x+3}{(x-1)(x^2+1)} dx$ or $\int \frac{x^4}{(x-1)(x^2+1)} dx$
8. Evaluate: $\int_0^{\pi} \frac{x}{1+sinx} dx$ or $\int_0^{\pi} \frac{xsinx}{1+cos^2x} dx$

Chapter 8
Application of integration

3 marks Questions

1. Find the area of the region bounded by y^2= x and x + y = 2
2. Find the area of the region bounded by $x^2 = 4y$ and x=4y-2
3. Find the area of the region bounded by $\{(x,y): x^2 \leq y \leq |x|\}$
4. Find the area of the smaller part of $x^2 + y^2 = 9$ cut off by x = $\frac{3}{\sqrt{2}}$
5. Find the area of the region bounded by y= $|x+3|$and x-axis between x=-6 and x=0

5 Marks questions

6. Find the area of the region bounded by $x^2 + y^2 = 16$,y = x and x axis in first quadrant
7. Find the area of the region bounded by $x^2 + y^2 = 16$,x = $\sqrt{3}y$ and x axis in first quadrant
8. Find the area of the region : $\{(x,y): x^2 + y^2 \leq 4, x + y \geq 2\}$
9. Find the area of the region : $\{(x,y): 9x^2 + y^2 \leq 36, 3x + y \geq 6\}$
10. Find the area of the region : $\{(x,y): 0 \leq x \leq y^2, 0 \leq x \leq y, 0 \leq y \leq 2\}$ or Find the area of the region : $\{(x,y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$

Chapter 9
Differential equations

Multiple choice questions

1. if m and n are the order and degree of the differential equation: $\frac{d}{dx}(\frac{d^2y}{dx^2}) = \sqrt{y}$ then m+n
   a) 3　　b) 4　　c) 2　　d) not defined
2. If m and n are the order and degree of the differential equation: $\frac{d}{dx}(\frac{dy}{dx}) = \sqrt{1 + (\frac{dy}{dx})^2}$
   a)3　　b) 4　　c) 2　　d) not defined
3. If m and n are the order and degree of the differential equation: $(\frac{dy}{dx})^2 + \frac{2x}{\frac{dy}{dx}} = 6y$
   a) 3　　b) 2　　c)4　　d) $not\ defined$
4. If $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + \sin(\frac{dy}{dx}) + 1 = 0$
   a)3　　b) 4　　c) 2　　d) not defined

5. Number of arbitrary constants in the general solution of differential equation of order 4 is
   a) 0 b) 2 c) 3 d) 4
6. Number of arbitrary constants in the particular solution of differential equation of order 4 is
   a) 0 b) 2 c) 3 d) 4
7. The general solution of the differential equation :$\frac{ydx-xdy}{y}$ = 0 is
   a) xy = C b) x = Cy^2 c) y = Cx d) $y = Cx^2$

2 mark questions

1. Find the general solution to the differential equations:$3e^x$tanydx+ (1- e^x)sec^2ydy=0
2. Find the general solution to the differential equations: xdy- ydx =$\sqrt{x^2 + y^2}$ dx
3. Find the general solution to the differential equation: $x\frac{dy}{dx} = y - xtan(\frac{y}{x})$
4. Find the general solution to the differential equation:$\frac{dy}{dx} + \frac{4x}{x^2+1}y + \frac{1}{(x^2+1)^2} = 0$

3 Marks questions

1. Find the particular solution to the differential equation:$(tan^{-1}y - x)dy = (1+y^2)dx$ given that when x=0,then y=0
2. Find the particular solution to the differential equation:$2ye^{\frac{x}{y}}dx+(y-2xe^{\frac{x}{y}})dy = 0$ given that x=0 then y=1
3. Find the particular solution to the differential equation:$x\frac{dy}{dx} + y$ = xcosx + sinx given that x=$\frac{\pi}{2}$ then y=1
4. Find the particular solutions to the differential equation: $(3xy + y^2)dx+(x^2 + xy)dy=0$ given that x=1 then y=1

Answers :-- Chapter 7. Integration

MCQ

1. b , $2\sqrt{x}$
   f(x)=$2\sqrt{x}$ +C
   f(1)=2=>$2\sqrt{1}$ +C=2
   C=2-2=0
2. c ,$\frac{\pi}{4}$
   $\int_0^{\frac{1}{\sqrt{2}}}\frac{1}{\sqrt{1-x^2}}dx=sin^{-1}x]_0^{\frac{1}{\sqrt{2}}} =sin^{-1}\frac{1}{\sqrt{2}} - sin^{-1}0 =\frac{\pi}{4} - 0=\frac{\pi}{4}$
3. b) 2
   $\int_0^a\frac{1}{4+x^2}dx = \frac{\pi}{8} => \frac{1}{2}tan^{-1}\frac{x}{2}]_0^a=\frac{\pi}{8} =>tan^{-1}\frac{a}{2} - tan^{-1}0 = \frac{\pi}{4} =>tan^{-1}\frac{a}{2} = = \frac{\pi}{4} =>\frac{a}{2}=\tan\frac{\pi}{4} =1$
4. a)xsinx
   First fundamental theorem in calculus if A(x) =$\int_0^a f(x)dx$ $then$ A'(x)=f(x)
5. b) $\frac{1}{x}$

$\int \frac{x-1}{x^2} e^x$dx =$\int (\frac{1}{x} - \frac{1}{x^2}) e^x$dx

$f(x) = \frac{1}{x}$ thenf'(x)=$- \frac{1}{x^2}$ integral of the form $\int (f(x) + f'(x)) e^x$dx=$e^x f(x) + c$

2 mark questions

1. $\int \frac{dx}{x+xlogx}$ =$\int \frac{dx}{x(1+logx)}$

Put t = 1+logx=> dt = $\frac{1}{x}$ dx

$$\int \frac{dx}{x(1+logx)} = \int \frac{1}{(1+logx)} \frac{dx}{x}$$

$$= \int \frac{1}{t} dt = log|t| + c$$

=log$|1 + logx|$+C ½+1+½

or

2. $\int \frac{sinx}{\sin(x+a)}$ dx=$\int \frac{\sin(t-a)}{\sin(t)}$ dx put t=x+a then dx=dt and x=t-a ½ mark

=$\int \frac{sint\ cosa - cost\ sina}{sint}$ dt

=$\int \frac{sint\ cosa}{sint} - \frac{cost\ sina}{sint}$ dtv

=$\int (cosa - sina \cot t)dt$ =cosa t- sina log$|sint|$+C 1 mark

= cosa (x+a)−sina log$|\sin(x+a)|$+C ½ mark

3. $\int \frac{dx}{2x^2-3x+7}\ dx$=

$\frac{1}{2} \int \frac{dx}{x^2 - \frac{3}{2}x + \frac{7}{2}}$

$= \frac{1}{2} \int \frac{dx}{(x-\frac{3}{4})^2 + -\frac{9}{16} + \frac{7}{2}}$

$= \frac{1}{2} \int \frac{dx}{\left(x-\frac{3}{4}\right)^2 + \frac{-9+56}{16}}$

$= \frac{1}{2} \int \frac{dx}{(x-\frac{3}{4})^2 + \frac{47}{16}}$ (1 mark)

$\frac{1}{2} \times \frac{4}{\sqrt{47}} tan^{-1}(\frac{x-\frac{3}{4}}{\frac{\sqrt{47}}{4}})$ +c

$= \frac{2}{\sqrt{47}} tan^{-1}(\frac{4x-3}{\sqrt{47}})$ + c (1mark)

4. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

=$\int \frac{dx}{\sqrt{-[-7+6x+x^2]}}$

=$\int \frac{dx}{\sqrt{-[x^2+6x-7]}}$

$=\int \frac{dx}{\sqrt{-[(x+3)^2-9-7]}}$

$=\int \frac{dx}{\sqrt{-[(x+3)^2-16]}}$ (1 mark)

$=\int \frac{dx}{\sqrt{16-(x+3)^2}}$

$=sin^{-1}\frac{x+3}{4}$ +C (1mark)

5. $put\ logx = t\ then\ x = e^t, dx = e^t dt$ ½ mark

$$\int [\frac{1}{logx} - \frac{1}{(logx)^2}]dx$$

$= \int [\frac{1}{t} - \frac{1}{t^2}]e^t dt$ ½ mark

$(x) = \frac{1}{t}$ then f'(x)=$-\frac{1}{t^2}$ integral of the form $\int(f(x) + f'(x))\, e^x$dx=$e^x f(x) + c$

$= \frac{1}{t}e^t$ +C= $\frac{1}{logx}e^{logx}$+C

$=\frac{x}{logx}$+C 1 mark

Or

$put\ logx = t\ then\ x = e^t, dx = e^t dt$

$\int \frac{logx}{(1+logx)^2}dx = \int \frac{t}{(1+t)^2}e^t dt$

$=\int \frac{1+t-1}{(1+t)^2}e^t dt$=$\int[\frac{1}{1+t} - \frac{1}{(1+t)^2}]e^t dt$

$=\frac{1}{1+logx}e^{logx}$ +C

$=\frac{x}{1+logx}$+C

6. x+2=0=>x=-2

$\int_{-5}^{5}|x+2|$dx=$\int_{-5}^{-2}|x+2|\,dx + \int_{-2}^{5}|x+2|\,dx$ property 2 ½ mark

$= \int_{-5}^{-2} -(x+2)\,dx + \int_{-2}^{5}(x+2)dx$ p ½ mark

$-\left\{\frac{1}{2}[x^2]_{-5}^{-2} + 2[x]_{-5}^{-2}\right\} + \frac{1}{2}[x^2]_{-2}^{5} + 2[x]_{-2}^{5}$

$-\ \{\frac{1}{2}[4-25] + 2\{-2--5]\}$+$\{\frac{1}{2}[25-4] + 2\{5--2]\}$

$\frac{21}{2}$ -6 +$\frac{21}{2}$ +14 =29 1 mark

OR

$$x^3 - x = 0 => x(x^2-1) = 0 => x = 0\ or\ x = -1\ or\ x = 1$$

$\int_{-2}^{1}|x^3 - x|$dx

=$\int_{-2}^{-1}|x^3-x|$dx+$\int_{-1}^{0}|x^3-x|$dx+$\int_{0}^{1}|x^3-x|$dx property 2

=$\int_{-2}^{-1} -(x^3 - x\ )$dx+$\int_{-1}^{0}(x^3-x)$dx+$\int_{0}^{1} -(x^3-x)$dx property 2

$=-\{\frac{1}{4}[x^4]_{-2}^{-1}-\frac{1}{2}[x^2]_{-2}^{-1}\}+\{\frac{1}{4}[x^4]_{-1}^{0}-\frac{1}{2}[x^2]_{-1}^{0}\}+-\{\frac{1}{4}[x^4]_{0}^{1}-\frac{1}{2}[x^2]_{0}^{1}\}$

$=-\{\frac{1}{4}[1-16]-\frac{1}{2}[1-4]\}+\{\frac{1}{4}[0-1]-\frac{1}{2}[0-1]\}-\{\frac{1}{4}[1-0]-\frac{1}{2}[1-0]\}$

$=-\{\frac{-15}{4}+\frac{3}{2}\}+\{-\frac{1}{4}+\frac{1}{2}\}-\{\frac{1}{4}-\frac{1}{2}\}$

$=\frac{15}{4}-\frac{3}{2}-\frac{1}{4}+\frac{1}{2}-\frac{1}{4}+\frac{1}{2}$

$=\frac{15-6-1+2-1+2}{4}=\frac{11}{4}$

7. Let $I=\int_0^{\frac{\pi}{2}}\frac{1}{1+\sqrt{\frac{sinx}{cosx}}}dx=\int_0^{\frac{\pi}{2}}\frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$------(1)

Apply Property P4, that x can be replaced bya-x that is $\frac{\pi}{2}$ - x

$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos(\frac{\pi}{2}-x)}}{\sqrt{\cos(\frac{\pi}{2}-x)}+\sqrt{\sin(\frac{\pi}{2}-x)}}\,dx=\int_0^{\frac{\pi}{2}}\frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\,dx$ ----(ii)

(i)+(ii) => $2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{cosx}+\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx=\int_0^{\frac{\pi}{2}}1\,dx=x]_0^{\frac{\pi}{2}}=\frac{\pi}{2}$

$I=\frac{\pi}{4}$

Or

Let $I=\int_3^5\frac{\sqrt{x}}{\sqrt{x}+\sqrt{8-x}}\,dx$------(1)

Apply propertyP3 x can be replaced by a+b-x that is 8 - x

$I=\int_3^5\frac{\sqrt{(8-x)}}{\sqrt{(8-x)}+\sqrt{(8-(8-x))}}\,dx=\int_3^5\frac{\sqrt{8-x}}{\sqrt{8-x}+\sqrt{x}}\,dx$ ----(ii)

(i)+(ii) => $2I=\int_3^5\frac{\sqrt{x}+\sqrt{8-x}}{\sqrt{x}+\sqrt{8-x}}\,dx=\int_3^5 1\,dx=x]_3^5=5-3$

I=1

$4\times\frac{1}{2}=2$ marks

8. $let\ I=\int_0^1\log(\frac{1-x}{x})dx$ ----(1)

Apply P4 , x can be replaced by a-x that is by 1- x

$I=\int_0^1\log(\frac{1-(1-x)}{1-x})dx\quad dx=\int_0^1\log(\frac{x}{1-x})dx$ ----(ii)

(i)+(ii) => $2I=\int_0^1(\log\frac{x}{1-x}+log\frac{1-x}{x})\,dx$

$=\int_0^1\log(\frac{x}{1-x}\times\frac{1-x}{x})dx$

$=\int_0^1\log(1)dx=0$

I=0

Or

$let\ I = \int_0^2 x\sqrt{2-x}\,dx$----(1)

Apply P4 , x can be replaced by a-x that is 2-x

I=$\int_0^2 (2-x)\sqrt{2-(2-x)}\,dx$

$$\mathrm{I} = \int_0^2 (2\sqrt{x} - x\sqrt{x})dx$$

=$\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\Big]_0^2$ =$\frac{16\sqrt{2}}{15}$

4×$\frac{1}{2}$ =2 marks

9. Let I = $\int_0^{\pi} \frac{e^{cosx}}{e^{cosx}+e^{-cosx}}\,dx$ (1)

Apply P4 , x can be replaced by a-x that is π-x

Let I = $\int_0^{\pi} \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)}+e^{-\cos(\pi-x)}}\,dx$=$\int_0^{\pi} \frac{e^{-cosx}}{e^{-cosx}+e^{cosx}}\,dx(1)$

Apply P4 , x can be replaced by a-x that is π-x

2I=$\int_0^{\pi} \frac{e^{cosx}+e^{-cosx}}{e^{cosx}+e^{-cosx}}\,dx$ =$\int_0^{\pi} 1dx$ =x$\Big]_0^{\pi}$ =π

I=$\frac{\pi}{2}$

4×$\frac{1}{2}$ =2 marks

Or $\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}$ dx multiply numerator and Denominator by$\sqrt{a-x}$

I=$\int_{-a}^{a}\sqrt{\frac{(a-x)^2}{a^2-x^2}}$ dx =$\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}$ =$\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}} - \int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}\,dx$

by property 7, $\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}\,dx = 0$ because $\frac{x}{\sqrt{a^2-x^2}}$ $is\ odd$

I=a $\sin^{-1}\frac{x}{a}\Big]_{-a}^{a}$ =a[$\frac{\pi}{2} - -\frac{\pi}{2}$] =aπ

4×$\frac{1}{2}$ =2 marks

3 Marks Questions

1. Let 5x-2=A(6x+2)+B

6A=5=> A=$\frac{5}{6}$ and 2A+B=-2 ==>2×$\frac{5}{6}$ +B=-2 => B=-2-$\frac{5}{3}$ =$\frac{-11}{3}$ 1mark

$$\int \frac{5x-2}{1+2x+3x^2}\,dx$$

$$= \int\left(\frac{\frac{5}{6}(2+6x)}{1+2x+3x^2} + \frac{\frac{-11}{3}}{1+2x+3x^2}\right)dx$$

$=\frac{5}{6}log\left|1+2x+3x^2\right|+\frac{-11}{9}\int\frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}$

$=\frac{5}{6}log\left|1+2x+3x^2\right|+\frac{-11}{9}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{-1}{9}+\frac{1}{3}}$ 1 mark

$=\frac{5}{6}log\left|1+2x+3x^2\right|+\frac{-11}{9}\int\frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}$

$=\frac{5}{6}log\left|1+2x+3x^2\right|+\frac{-11}{9}\times\frac{\sqrt{2}}{3}\tan^{-1}\frac{3x+1}{\sqrt{2}}+C$ 1 mark

2.

Let 3x +5 = A(2x-8) + B

3x+5=2Ax-8A+B

2A=3=>A=$\frac{3}{2}$ and -8A+B=5 =>-8×$\frac{3}{2}$ +B=5 =>B=5+12=17 1mark

$\int\frac{3x+5}{\sqrt{x^2-8x+7}}dx$

$=\int\left(\frac{\frac{3}{2}(2x-8)}{\sqrt{x^2-8x+7}}+\frac{17}{\sqrt{x^2-8x+7}}\right)dx$

$=3\sqrt{x^2-8x+7}+17\int\frac{dx}{\sqrt{(x-4)^2+-16+7}}$

$=3\sqrt{x^2-8x+7}+17\int\frac{dx}{\sqrt{(x-4)^2-9}}$ 1 mark

$3\sqrt{x^2-8x+7}+17log\left|x-4+\sqrt{x^2-8x+7}\right|$ +C 1 mark

3. $\frac{1-x^2}{x(1-2x)}=A+\frac{B}{x}+\frac{C}{1-2x}$

$1-x^2$=A$x(1-2x)$+ B(1-2x)+cx

$x=0$ => B=1 and x=$\frac{1}{2}$ =>c=$\frac{3}{2}$ comparing coeff. of x^2 we get A=$\frac{1}{2}$ 1 mark

$\int\frac{1-x^2}{x(1-2x)}dx=\int\left(\frac{1}{2}+\frac{1}{x}+\frac{\frac{3}{2}}{1-2x}\right)dx$ 1 mark

$=\frac{1}{2}x+log|x|-\frac{3}{4}l\ og|1-2x|$+C 1 mark

4. $\int\frac{7x^6dx}{7x^7(x^7+5)}$ multiply numerator and denominator by $7x^6$

$put\ x^7=t\ then\ 7x^6dx=dt,$

$$\int\frac{7x^6dx}{7x^7(x^7+5)}=\frac{1}{7}\int\frac{dt}{t(t+5)}$$

$\frac{1}{t(t+5)}=\frac{A}{t}+\frac{B}{t+5}$ =>1=A(t+5)+Bt ,t=0 A=$\frac{1}{5}$ and t=-5 then B=$\frac{-1}{5}$ 1 mark

$\frac{1}{7}\int\frac{dt}{t(t+5)}=\frac{1}{7}\left[\frac{1}{5}\int\frac{dt}{t}+\frac{-1}{5}\int\frac{dt}{t+5}\right]$ 1 mark

$=\frac{1}{35}$[log$|t|$- log$|t+5|$+C

$=\frac{1}{35}$[log$\left|\frac{t}{t+5}\right|$+C

$=\frac{1}{35}$[log$\left|\frac{x^7}{x^7+5}\right|$+C 1 mark

or $\int\frac{dx}{e^x-1}$

$=\int \frac{e^x dx}{e^x(e^x-1)}$ multiplying numerator and denominator by e^x

$put\ e^x = t\ then\ e^x dx = dt,$

$\int \frac{e^x dx}{e^x(e^x-1)}$ $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$$= \int \frac{dt}{t(t-1)}$$

1=A(t-1)+Bt ,t=0 A=-1and t=1 then B=1 1 mark

$=\int \frac{dt}{t(t-1)}$

$= -\int \frac{dt}{t} + \int \frac{dt}{t-1}$ 1 mark

= [-log$|t|$+log$|t-1|$]+C

= [log$\left|\frac{t-1}{t}\right|$+C= log$\left|\frac{e^x-1}{e^x}\right|$+C 1 mark

5. Evaluate:$\int \frac{2x-1}{(x-1)(x-2)(x-3)}\ dx$

Let $\frac{2x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{x-2} + \frac{C}{x-3}$

2x-1=A$(x-2)(x-3) + B(x-1)(x-3) + c(x-1)(x-2)$

Put x=1 ,A=$\frac{1}{2}$put x=2 get B=-3 ,put x=3 getc=$\frac{5}{2}$ 1mark

$\int \frac{2x-1}{(x-1)(x-2)(x-3)}\ dx$

$=\int \frac{1/2}{(x-1)} + \frac{-3}{x-2} + \frac{5/2}{x-3}$dx 1 mark

=$\frac{1}{2}$log$|x-1|$-3 log$|x-2|$ +$\frac{5}{2}$ log$|x-3|$+C 1 mark

or

Let $\frac{2x-1}{(x-1)^2(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

2x-1=A$(x-1)(x-2) + B(x-2) + c(x-1)^2$

Put x=1 ,B=-1put x=2 get C=3 ,compare coeff.x^2 getA+C=0=>A=-C=>A=-3 1mark

$\int \frac{2x-1}{(x-1)^2(x-2)}dx$

$=\int \frac{-3}{(x-1)} + \frac{-1}{(x-1)^2} + \frac{3}{x-2}$dx 1 mark

=-3log$|x-1|$+$\frac{1}{(x-1)}$ +3 log$|x-2|$+C 1 mark

6. Put $x^2 = t\ then\ 2xdx = dt$

$\int \frac{2x}{(x^2+1)(x^2+4)}dx$

$=\int \frac{1}{(t+1)(t+4)}dt$ Let $\frac{1}{(t+1)(t+4)} = \frac{A}{(t+1)} + \frac{B}{t+4}$

$=\int [\frac{\frac{1}{3}}{(t+1)} + \frac{-\frac{1}{3}}{t+4}]$dt 1=A$(t+4) + B(t+1)$

$=\frac{1}{3}\log|t+1| - \frac{1}{3}\log|t+4| + C$ Put t=-1 ,A=$\frac{1}{3}$ & put t=-4 get B=$\frac{-1}{3}$ 1mark

$= \frac{1}{3}\log|x^2+1| - \frac{1}{3}\log|x^2+4|$+C 1 mark

or $\int \frac{1}{(x^2+1)(x^2+4)}dx$

Put $x^2 = y$

$\int \frac{1}{(y+1)(y+4)}dx = \int \frac{1}{(y+1)(y+4)}dx$

$Let \frac{1}{(y+1)(y+4)} = \frac{A}{(y+1)} + \frac{B}{y+4}$

1=A$(y+4)+B(y+1)$

Put y=-1 ,A=$\frac{1}{3}$put y=-4 get B=$\frac{-1}{3}$ 1mark

$\int \frac{1}{(y+1)(y+4)}\ dx$

$=\int\{\frac{\frac{1}{3}}{(y+1)} + \frac{-\frac{1}{3}}{y+4}]$dx

$= \int\{\frac{\frac{1}{3}}{(x^2+1)} + \frac{-\frac{1}{3}}{x^2+4}]$dx 1 mark

$=\frac{1}{3}\tan^{-1}x - \tan^{-1}\frac{x}{2}$+C 1 mark

7. $\int \frac{2x+3}{(x-1)(x^2+1)}dx$

$$Let \frac{2x+3}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+c}{(x^2+1)}$$

$\int \frac{2x+3}{(x-1)(x^2+1)}$ dx

2x+3=A(x^2+1)+(Bx+C)(x-1)

X=1 then A= $\frac{5}{2}$,x=0 then 3=A-C=>C= -$\frac{1}{2}$

$= \int \frac{\frac{5}{2}}{x-1} + \frac{-\frac{5}{2}x-\frac{1}{2}}{(x^2+1)}$ dx

equating coeff.x^2we get A+B=0 hence B=-$\frac{5}{2}$

1 mark

$=\frac{5}{2}\int \frac{1}{x-1}dx - \frac{5}{4}\int \frac{2x}{(x^2+1)}dx - \frac{1}{2}\int \frac{1}{(x^2+1)}dx$ 1 mark

$= \frac{5}{2}\log|x-1|-\frac{5}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x$+C 1 mark

or $\int \frac{x^4-1+1}{(x-1)(x^2+1)}dx$

$=\int \frac{x^4-1}{(x-1)(x^2+1)} + \frac{1}{(x-1)(x^2+1)}dx$

$=\int \frac{(x^2+1)(x-1)(x+1)}{(x-1)(x^2+1)}dx + \int \frac{1}{(x-1)(x^2+1)}dx$

$= \int (x+1)dx + \int \frac{1}{(x-1)(x^2+1)} dx$

Let $\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+c}{(x^2+1)}$

1=A(x^2+1)+(Bx+C)(x-1)

X=1 then A=$\frac{1}{2}$,x=0 then 1=A-C=>C= -$\frac{1}{2}$

equating coeff.x^2we grt A+B=0 hence B=-$\frac{1}{2}$ 1 mark

$\int (x+1)dx + \int \frac{1}{(x-1)(x^2+1)} dx$

$= x^2 + x + \int \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}x-\frac{1}{2}}{(x^2+1)} dx$

$= x^2 + x + \frac{1}{2}\int \frac{1}{x-1} dx - \frac{1}{4}\int \frac{2x}{(x^2+1)} dx - \frac{1}{2}\int \frac{1}{(x^2+1)} dx$ 1 mark

$= x^2 + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1} x + C$

8. $I = \int_0^{\pi} \frac{x}{1+sinx} dx$ …..(1)

Apply P4 that is replace x byπ-x

$I = \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)} dx$

$= \int_0^{\pi} \frac{\pi - x}{1+\sin x} dx$ …..(2) ½ mark

(1)+(2)=>

$2I = \int_0^{\pi} \frac{x+\pi-x}{1+\sin x} dx$

$= \pi \int_0^{\pi} \frac{1}{1+sinx} dx$

$= \pi \int_0^{\pi} \frac{1-sinx}{(1+sinx)(1-sinx)} dx$ ½ mark

$= \pi \int_0^{\pi} \frac{1-sinx}{(1-sin^2 x)} dx$

$= \pi \int_0^{\pi} \frac{1-sinx}{cos^2 x} dx$

$= \pi \int_0^{\pi} (sec^2 x - secxtanx) dx$ 1 mark

$\pi[\tan x - \sec x]_0^{\pi}$

$= \pi$ (0- -1)-(0-1)=2 π =>I= π 1 mark

or

9. $I = \int_0^{\pi} \frac{xsinx}{1+\cos^2 x} dx$ …..(1)

Apply P4 that is replace x byπ-x

$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$

$= \int_0^{\pi} \frac{(\pi - x)sinx}{1+\cos^2 x} dx$ …..(2) ½ mark

(1)+(2)=>

$2I = \int_0^{\pi} \frac{xsinx + (\pi - x)sinx}{1+\cos^2 x} dx$

$= \pi \int_0^{\pi} \frac{sinx}{1+\cos^2 x} dx$ ½ mark

Put cosx=t then -sinxdx =dt

$= \pi\int_1^{-1} \frac{-dt}{1+t^2}$ x=o then t=1 and x= π then t=-1 ½ mark

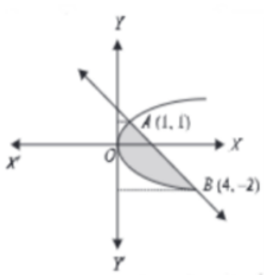
$= -\pi[\tan^{-1} t]_1^{-1}$ 1 mark

$= -\pi[-\frac{\pi}{4}-\frac{\pi}{4}] = -\pi[-\frac{\pi}{2}] \Rightarrow I = \pi^2/2$ ½ mark

Answers:-- Chapter 8. Applications of integration

3 mark questions

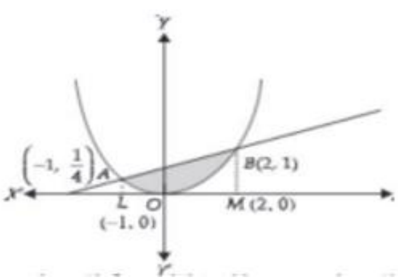
1



Required area $=\int_{-2}^{1}(2-y-y^2)$dy

$=\frac{9}{2}$ sq units

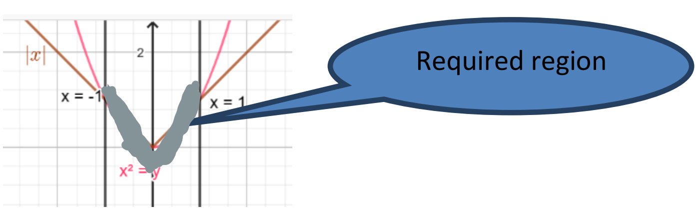
2



Required area$=\int_{-1}^{2}[\frac{x+2}{4}-\frac{x^2}{4}]dx$

$=\frac{9}{8}$ sq units

3

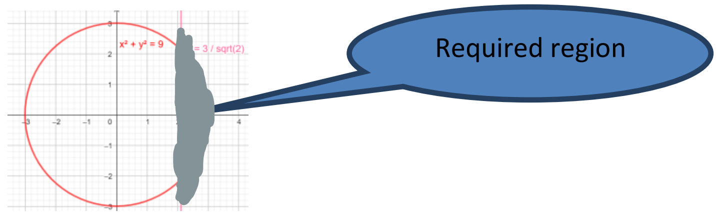


Region is symmetric required area=2 times area in right half plane

Required area=2 $\int_0^1[x-x^2]dx$

$=\frac{1}{3}$ sq units

4



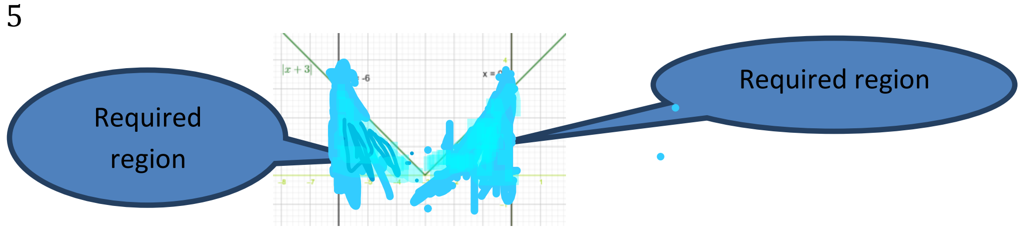
Region is symmetric about x axis hence required area =2 area in upper half plane

Required area $=\int_{\frac{3}{\sqrt{2}}}^{3} 2\sqrt{9-x^2}\,dx$

$=2[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}]_{\frac{3}{\sqrt{2}}}^{3}$

$=\frac{9}{4}\pi-9$ sq units

5

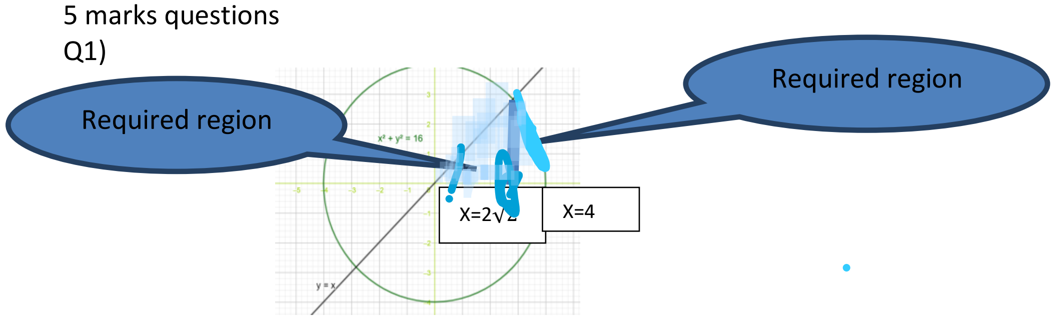


Required area $=\int_{-6}^{0}|x+3|dx=\int_{-6}^{-3}-(x+3)dx+\int_{-3}^{0}(x+3)dx$

= 9 sq units

5 marks questions

Q1)



Solving $x^2+y^2=16$ and y=x we get x= $\pm2\sqrt{2}$

Required area= $\int_{0}^{2\sqrt{2}} x\,dx+\int_{2\sqrt{2}}^{4}\sqrt{16-x^2}\,dx$

$=\frac{1}{2}[x^2]_{0}^{2\sqrt{2}}+[\frac{x}{2}\sqrt{16-x^2}+8\sin^{-1}\frac{x}{4}]_{2\sqrt{2}}^{4}$

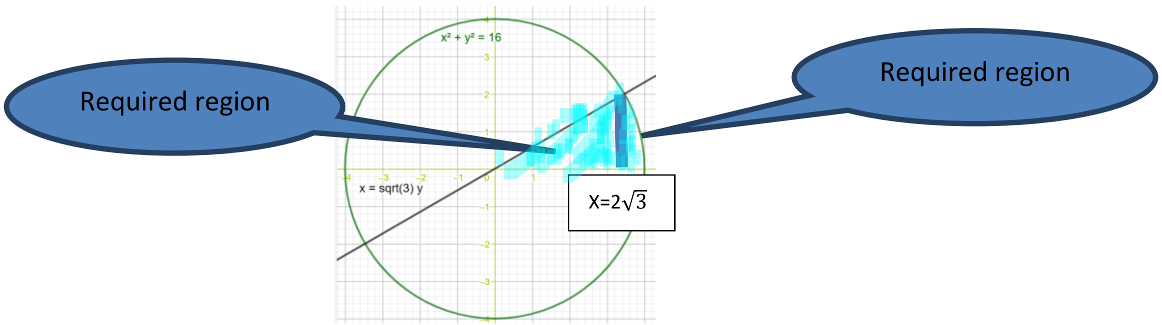
=2πsq units

Q2) Solving $x^2+y^2=16$ and $y=\frac{x}{\sqrt{3}}$ we get x= $\pm2\sqrt{3}$

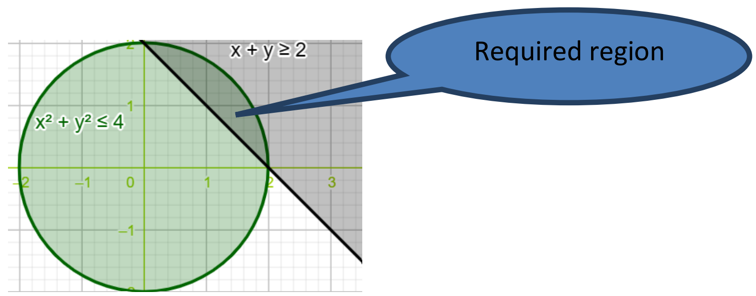
Required area= $\int_0^{2\sqrt{3}} x\,dx + \int_{2\sqrt{3}}^{4} \sqrt{16-x^2}\,dx$

$=\frac{1}{2}[x^2]_0^{2\sqrt{3}} + [\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\frac{x}{4}]_{2\sqrt{3}}^{4}$

$=\frac{4\pi}{3}$sq units



Q3)



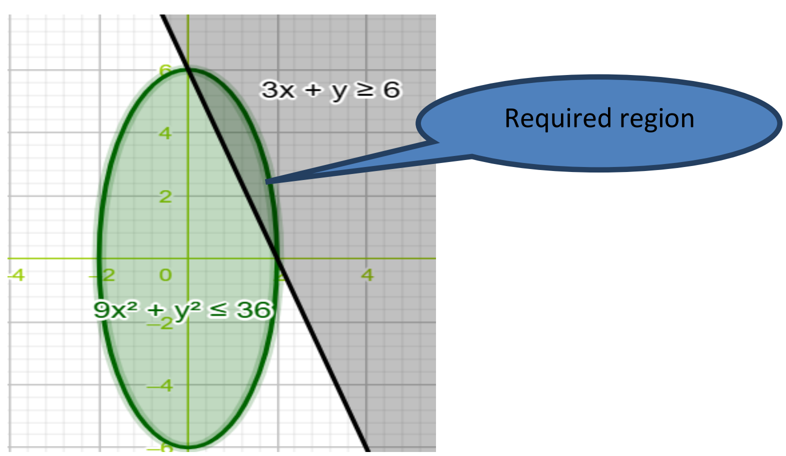
Required area= $\int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$ -

$= [\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}]_0^2 - [2x-\frac{x^2}{2}]_0^2$

$=(0-0)+2(\frac{\pi}{2}-0)-[4-2]+(0-0)$

$=(\pi-2)$sq units

Q4)



Required area= $\int_0^2 3\sqrt{4-x^2}\,dx - \int_0^2 (6-3x)\,dx$ -

$= 3[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}]_0^2 - [6x-\frac{3x^2}{2}]_0^2$

$=3\{(0-0)+2(\frac{\pi}{2}-0)\}-[12-6]+(0-0)$

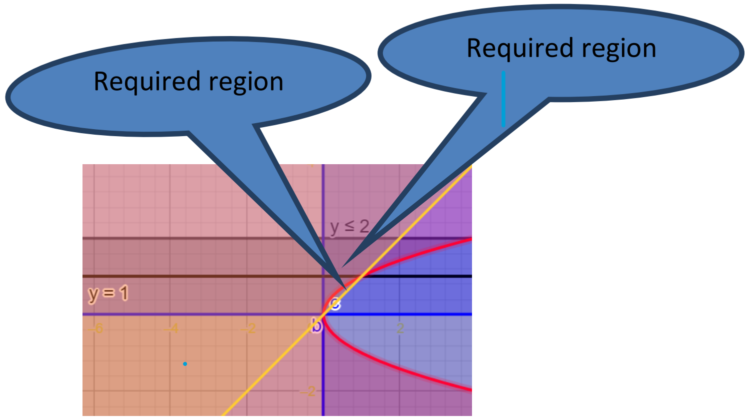
=(3π-6)sq units

Q5) eliminating x from y^2=x and y=x we get y=0 0r y=1

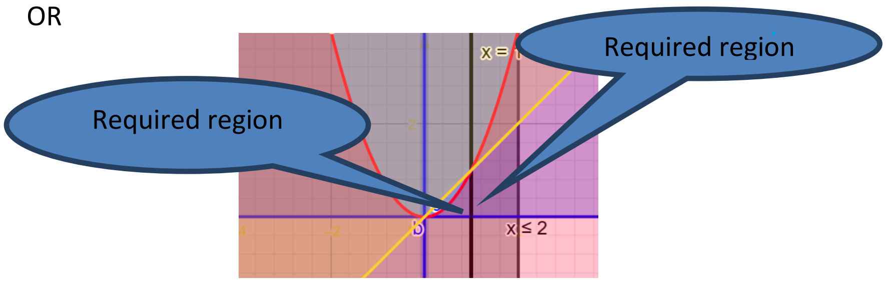
Required area =$\int_0^1 y^2$ dy+$\int_1^2 y$ dy

=$\frac{1}{3}y^3]_0^1 + \frac{1}{2}y^2]_1^2$

=$\frac{1}{3}+\frac{3}{2}=\frac{11}{6}$ sq units



OR



eliminating y from x^2= y and y=x we get x=0 0r x=1

Required area =$\int_0^1 x^2$ dx+$\int_1^2 x$ dx

=$\frac{1}{3}x^3]_0^1 + \frac{1}{2}x^2]_1^2$

=$\frac{1}{3}+\frac{3}{2}=\frac{11}{6}$ sq units

For question numbers 1 to 5 each step carries 1 mark each

Chapter 9

Differential equations

Answers to MCQ

1. (b) order 3 degree 1
2. (b) order 2 degree 2
3. (c)order1 degree 3
4. (d) degree not defined
5. (d) order= number of arbitrary constants in general solution
6. (a) 0
7. (c),y=cx

Answers to 2 marks questions

1. $3e^x$tanydx=- (1- e^x)sec^2ydy

$\int\frac{3e^x\mathrm{dx}}{e^x-1}=\int\frac{sec^2\mathrm{ydy}}{tany}$ put $e^x-1=p\ and\ tany=q$

$=> e^x dx = dp\ and\ \sec^2 y\, dy = dq$ 1 mark

$\int \frac{3dp}{p} = \int \frac{dq}{q}$

=>3log|p|=log|q|+C

=>3log|$e^x - 1$|=log|$tany$|+C 1 mark

2. xdy- ydx =$\sqrt{x^2 + y^2}$ dx divide by xdx

$\frac{dy}{dx} - \frac{y}{x} = \frac{\sqrt{x^2+y^2}}{x}$ put y=vx then $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 mark

$v + x\frac{dv}{dx}$ -v=$\sqrt{1 + v^2}$

=>$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ integrating both sides ½ mark

=> $log\left|v + \sqrt{1 + v^2}\right|$=log|$x$|+C

=> $log\left|\frac{y+\sqrt{x^2+y^2}}{x}\right|$= log|$x$|+C ½ mark

3. $\frac{dy}{dx} = \frac{y}{x} - tan\left(\frac{y}{x}\right)$ put y=vx then $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 mark

$v + x\frac{dv}{dx}$ =v-tanv

=>$-\int cotv\ dv = \int \frac{dx}{x}$

=>-log$\left|\sin(\frac{y}{x})\right|$= log|$x$|+C 1 mark

4. Linear in y, $\frac{dy}{dx}$ +py= Q where $\frac{4x}{x^2+1}$ and Q=$\frac{-1}{(x^2+1)^2}$

I F =$e^{\int\frac{4x}{x^2+1}dx}$=$e^{2\log(x^2+1)}$=$(x^2+1)^2$ 1 mark

Solution y($(x^2+1)^2$=$\int \frac{-1}{(x^2+1)^2}(x^2+1)^2$dx

=> y$(x^2+1)^2$=x+C 1 mark

3 marks questions

1. Linear differential equation in x , $\frac{(tan^{-1}y - x)}{(1+y^2)} = \frac{dx}{dy}$

=> $\frac{(tan^{-1}y}{(1+y^2)}$ - $\frac{x}{(1+y^2)} = \frac{dx}{dy}$

=>$\frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{(tan^{-1}y}{(1+y^2)}$ 1 mark

I.F.=$e^{\int\frac{1}{(1+y^2)}}$ =$e^{\tan^{-1}y}$ ½ mark

Solution is x$e^{\tan^{-1}y} = \int e^{\tan^{-1}y}\frac{(tan^{-1}y}{(1+y^2)}$dy ½ mark

Put $\tan^{-1}y = t\ then \frac{1}{(1+y^2)}$dy =dt

=$\int e^{\tan^{-1}y}\frac{(tan^{-1}y}{(1+y^2)}$dy

=$\int te^t$dt= $te^t - e^t$ +C

solution is x$e^{\tan^{-1}y} = \tan^{-1}y\, e^{\tan^{-1}y}$ - $e^{\tan^{-1}y} + C$

=>x= $\tan^{-1}y - 1 + c\ e^{-\tan^{-1}y}$ 1 mark

2. Puy x=uy then$\frac{dx}{dy}$ =u +y$\frac{du}{dy}$ ½ mark

$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}$

=>$u + y\frac{du}{dy} = \frac{2ue^u-1}{2e^u}$

=> $y\frac{du}{dy} = \frac{2ue^u-1}{2e^u}$ -u

=> $y\frac{du}{dy} = \frac{2ue^u-1-2u2e^u}{2e^u}$ 1 mark

$\int 2e^u du = -\int \frac{dy}{y}$ =>$2e^u = -log|y|$ +C

=>$2e^{\frac{x}{y}} = -log|y|$ +C 1mark

$2e^{\frac{x}{y}} + log|y|$ =2 ½ mark

3. $x\ \frac{dy}{dx} + y = \text{xcosx} + \text{sinx}$

$Linear\ in\ y, \frac{dy}{dx} + \frac{y}{x} = \frac{xcosx+sinx}{x}$ ½ mark

$I.F = e^{\int\frac{dx}{x}} = e^{logx} = x$ 1 mark

$solution : y \times x = \int \frac{xcosx + sinx}{x} \times xdx$

$xy = \int xcosxdx + \int sinxdx$=>

$xy = x\int cosxdx - \int [\frac{dx}{dx}\int cosxdx]dx + \int sinxdx$

xy=xsinx-$\int sinxdx + \int sinxdx$=

xy=xsinx+c 1 mark

y=sinx when x= $\frac{\pi}{2}$ and y=1 then c=0 ½ mark

4. $\frac{dy}{dx} = \frac{-(3xy+y^2)}{(x^2+xy)}$ put y=vx then $\frac{dy}{dx} = v + x\frac{dv}{dx}$ ½ mark

$v + x\frac{dv}{dx} = \frac{-(3v+v^2)}{(1+v)}$

$x\frac{dv}{dx} = \frac{-(3v+v^2)}{(1+v)} - v$

$x\frac{dv}{dx} = \frac{-3v - v^2 - v - v^2}{(1+v)}$

$x\frac{dv}{dx} = \frac{-4v - 2v^2}{(1+v)}$

$x\frac{dv}{dx} = \frac{-2(2v + v^2)}{(1+v)}$

$\frac{(1+v)dv}{2v+v^2} = \frac{-2dx}{x}$ 1 mark

$\int \frac{(1+v)dv}{2v+v^2}$=-2$\int \frac{dx}{x}$

put $2v + v^2 = t\ \ then(2 + 2v)dv$=dt=>2(1+v)dv=dt

$\frac{1}{2}\int \frac{dt}{t}$=-2$\int \frac{dx}{x}$

=>$\frac{1}{2}$log$|t|$=-2log$|x|$+c

=>log$|t|$=-4log$|x|$+2c
=>$|2\text{v}+v^2|$+4log$|x|$=2c

=> log$\left|\left[2\frac{y}{x}+\left(\frac{y}{x}\right)^2\right]x^4\right|$=2c 1 mark
=>2x^{3}y+x$^2y^2$=$e^{\pm 2c}$
=>2x^{3}y+x$^2y^2$=K given x=1 and y=1 then k=3
=>2x^{3}y+x$^2y^2$=3 ½ mark

10. VECTOR ALGEBRA

MCQ

1

Find the vector components of the vector with initial point (2,1) and terminal point (-5,7)

a) 2 $\hat{\imath}$, $\hat{\jmath}$ b) -7 $\hat{\imath}$, 6 $\hat{\jmath}$ c) -5 $\hat{\imath}$, $\widehat{7\jmath}$ d) 3 $\hat{\imath}$, 6 $\hat{\jmath}$

2

Find the vector in the direction of 5 $\hat{\imath}$ - $\hat{\jmath}$ +2 $\hat{k}$ which has magnitude 8 units

a) $\frac{5\hat{i}-\hat{j}+2\hat{k}}{30}$ b) $\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{30}}$ c) $\frac{8(5\hat{i}-\hat{j}+2\hat{k})}{\sqrt{30}}$ d) $8(5\hat{i}-\hat{j}+2\hat{k})$

3

The value of $\hat{\imath}.(\hat{\jmath} \text{ x } \hat{k}) + \hat{\jmath}.(\hat{\imath} \text{ x } \hat{k}) + \hat{k}.(\hat{\imath} \text{ x } \hat{\jmath})$ is

b) 0 b) -1 c) 1 d) 3

4

If $\vec{a} = 2\,\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}$, $\vec{b} = -\,\hat{\imath} + 2\,\hat{\jmath} + \hat{k}$, $\vec{c} = 3\,\hat{\imath} + \hat{\jmath}$ are such that $\vec{a}$ +λ $\vec{b}$ is perpendicular to $\vec{c}$,then find the value of λ.

a) 0 b) -8 c) 1 d) 8

5

Find the projection of the vector $\vec{a} = 2\,\hat{\imath} + 3\,\hat{\jmath} + 2\,\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\,\hat{\jmath} + \hat{k}$

a) 0 b) 10 c) $\sqrt{6}$ d) $\frac{10}{\sqrt{6}}$

2 MARKS

6 If A(1,2,3) , B(-1,0,0) and C(0,1,2) are the vertices of a triangle ABC, Find angle ABC

7

If $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$, $\vec{b} = -\,\hat{\imath} + 2\,\hat{\jmath} + 2\,\hat{k}$ and $\vec{c} = -\,\hat{\imath} + 2\,\hat{\jmath} - \hat{k}$ are three vectors then find a unit vector normal to the vectors $(\vec{a} + \vec{b})$ and $(\vec{b} - \vec{c})$

8 Fid the area of the parallelogram having vertices A,B,C and D with position vectors $\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $2\hat{i}+2\hat{j}-\hat{k}$ respectively

9

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$,then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$

10

If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \text{ x } \vec{b} = 3\,\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\vec{a} \text{ and } \vec{b}$

3 MARKS

11 Let $\vec{a} = \hat{\imath} + 4\,\hat{\jmath} + 2\,\hat{k}$, $\vec{b} = 3\,\hat{\imath} - 2\,\hat{\jmath} + 7\,\hat{k}$ and $\vec{c} = 2\,\hat{\imath} - \hat{\jmath} + 4\,\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a} \text{ and } \vec{b}$ and $\vec{c}\,.\vec{d} = 15$

12

The scalar product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to one. Find the value of λ.

13

Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two find $|\vec{a} + \vec{b} + \vec{c}|$

14

If with reference to the right handed system of mutually perpendicular unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$, $\vec{\alpha} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{\beta} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ then express $\vec{\beta}$ in the form $\overrightarrow{\beta} = \overrightarrow{\beta_1} + \overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$

15

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$

11 THREE DIMESIONAL GEOMETRY

MCQ

1 The equation of a line is $\frac{x-1}{-2} = \frac{y+3}{3} = \frac{z+2}{6}$, find the direction cosines of a line parallel to the given line.

(a) -2/7, 3/7, 6/7 (b) 2/7, 3/7, 6/7 (c) -2 , 3 , 6 (d) 2 , -3 , -6

2 If the cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write its vector equation.

(a) $\vec{r} = (\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \mu\,(5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$
(b) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu\,(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$
(c) $\vec{r} = (3\hat{\imath} + 4\hat{\jmath} + 3\hat{k}) + \mu\,(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$

(d) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu\,(5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$

3 If a line makes angles 90^0 and 60^0 with the positive direction of x and y axes ,find the angle which it makes with positive direction of z -axis.

(a) π/3 (b) π/4 (c) π/6 (d) 0

4 Write direction cosines of a line parallel to z-axis.

(a) 1,0,0 (b) 0,0,1 (c) 1,1,0 (d) -1,-1,-1

5 Find the foot of the perpendicular drawn from the point (2,-3,4) on the y-axis.

(a) (2,0,4) (b) (0.3.0) (c) (0,-3,0) (d) (-2,0,-4)

2 MARKS

6 Find the value of k so that the lines x = -y = kz and x – 2 = 2y + 1 = -z + 1 are perpendicular to each other.

7 Find the vector and Cartesian equation of the line passing through points (3, -2, -5) and ,(5, -4, 6)

8 The x –coordinate of a point on the line joining the points P (2, 2, 1) and Q (5, 1, -2) is 4 . Find its z- co ordinate

9 The points A(1,2,3), B(-1,-2,-3)and C(2,3,2) are the vertices of a parallelogram ,then find the equation of CD.

10 Show that the line through the points A(4,7,8) and B(2,3,4) is parallel to the line through the points C(-1,-2,1) and D (1,2,5)

3 MARKS

11 Find the value of ,λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other

12 Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$
$$\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$$

13 Find the equation of line passing through (1,2,3) and midpoint of the line joining (2,-1,3) and (1,2,5)

14 Prove that the line through A(0,-1,-1) and B(4,5,1) intersects the line through C(3,9,4) and D(-4,4,4)

15 Find the distance of the point (– 2, 4, – 5) from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$

5 MARKS

16 Find the co-ordinates of the foot of the perpendicular drawn from the point A (1,8,4) to the line joining B (0, -1,3) and C (2,-3,-1).
Also find the length of the perpendicular ,equation of the perpendicular line and image of the point with respect to the line BC .

17 Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also, find the point of intersection

18 Find the equation of the line passing through the point P (-1,3,-2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

19 Find the equation of the line passing through the point P (2, -1,3) and perpendicular to the lines $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda\left(2\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{\imath} - \hat{\jmath} - 3\hat{k}\right) + \mu\left(\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right)$

20 An insect is crawling along the line $\vec{r} = \left(6\hat{\imath} + 2\hat{\jmath} + 2\hat{k}\right) + \lambda\left(\hat{\imath} - 2\hat{\jmath} + 2\hat{k}\right)$ and another insect is crawling along the line $\vec{r} = \left(-4\hat{\imath} - \hat{k}\right) + \mu\left(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k}\right)$. At what points on the lines should they reach so that the distance between them is the shortest. Find the shortest possible distance between them.

MINIMUM LEVEL LEARNING

	VECTOR ALGEBRA	
	MCQ	
1	b) --7 $\hat{\imath}$, 6 $\hat{\jmath}$	
2	c) $\dfrac{8\left(5\,\hat{i} - \hat{j} + 2\,\hat{k}\right)}{\sqrt{30}}$	
3	c) 1	
4	d) 8	
5	d) $\dfrac{10}{\sqrt{6}}$	
	2 Marks	
6	$\cos\theta = \dfrac{\overrightarrow{BC} \quad . \quad \overrightarrow{BA}}{\left\lvert\overrightarrow{BC}\right\rvert \quad \left\lvert\overrightarrow{BA}\right\rvert}$ $= \dfrac{\left(2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}\right) \quad . \quad \left(\hat{i} + \hat{j} + 2\,\hat{k}\right)}{\left\lvert\left(2\,\hat{i} + 2\,\hat{j} + 3\,\hat{k}\right)\right\rvert \quad \left\lvert\left(\hat{i} + \hat{j} + 2\,\hat{k}\right)\right\rvert} = \dfrac{10}{\sqrt{102}}$ $\theta = \cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$	
7	$\vec{a} + \vec{b} = 3\hat{\jmath} + \hat{k}$ $\vec{b} - \vec{c} = 3\hat{k}$ $\left(\vec{a} + \vec{b}\right) \times \left(\vec{b} - \vec{c}\right) = 9\hat{\imath}$ $\left\lvert\left(\vec{a} + \vec{b}\right) \times \left(\vec{b} - \vec{c}\right)\right\rvert = 9$ $Required \quad unit \quad normal \quad vector \quad is \dfrac{\left(\vec{a} + \vec{b}\right) \times \left(\vec{b} - \vec{c}\right)}{\left\lvert\left(\vec{a} + \vec{b}\right) \times \left(\vec{b} - \vec{c}\right)\right\rvert} = \hat{\imath}$	
8	$\overrightarrow{AB} = \hat{j} + 2\hat{k}$ and $\overrightarrow{AC} = \hat{i} + 2\vec{j}$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix} = -4\hat{\imath} + 2\hat{\jmath} - \hat{k}$ Area of parallelogram = $\left\lvert\overrightarrow{AB} \times \overrightarrow{AC}\right\rvert = \sqrt{21}$	
9	Given $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = \lvert\vec{c}\rvert = 1, \vec{a} + \vec{b} + \vec{c} = \vec{0}$	

	$\left(\vec{a}+\vec{b}+\vec{c}\right) \;.\; \left(\vec{a}+\vec{b}+\vec{c}\right)=0$ $\lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+\lvert\vec{c}\rvert^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$ $\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=\frac{-3}{2}$	
10	$\vec{a} \times \vec{b} = 3\,\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ $\quad \lvert\vec{a}\times\vec{b}\rvert = 7$ $\lvert\vec{a}\times\vec{b}\rvert = \lvert\vec{a}\rvert\lvert\vec{b}\rvert \sin\theta \Rightarrow 7 = 2\times 7\sin\theta \Rightarrow \sin\theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{6}$	
	3 Marks	
11	$\vec{d} = \lambda\left(\vec{a}\times\vec{b}\right) = \lambda\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 4 & 2\\ 3 & -2 & 7\end{vmatrix} = \lambda\left(32\hat{\imath}-\hat{\jmath}-14\hat{k}\right)$ $\vec{c}.\vec{d} = 15 \Rightarrow \lambda = \frac{5}{3}$ $\vec{d} = \frac{5}{3}\left(32\hat{\imath}-\hat{\jmath}-14\hat{k}\right)$	
12	$\vec{a} = \vec{\imath} + \hat{\jmath} + \hat{k}$ $\quad \vec{b} = 2\,\hat{\imath} + 4\,\hat{\jmath} - 5\,\hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 2\,\hat{\jmath} + 3\,\hat{k}$ $\vec{b}+\vec{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k} \Rightarrow \lvert\vec{b}+\vec{c}\rvert = \sqrt{(2+\lambda)^2+36+4}$ Unit vector along sum = $\frac{\vec{b}+\vec{c}}{\lvert\vec{b}+\vec{c}\rvert} = \frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+36+4}}$ Given $\vec{a} \;.\; \frac{\vec{b}+\vec{c}}{\lvert\vec{b}+\vec{c}\rvert} = 1 \Rightarrow (\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}) \;.\; \left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+36+4}}\right) = 1$ Solving $\lambda = 1$	
13	Given $\vec{a}.\left(\vec{b}+\vec{c}\right)=\vec{0}$, $\vec{b}.\left(\vec{a}+\vec{c}\right)=\vec{0}$ $\quad \vec{c}.\left(\vec{a}+\vec{b}\right)=\vec{0}$ $\lvert\vec{a}+\vec{b}+\vec{c}\rvert^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right) = \vec{a}.\vec{a}+\vec{a}.\left(\vec{b}+\vec{c}\right)+\vec{b}.\vec{b}+\vec{b}.\left(\vec{a}+\vec{c}\right)+\vec{c}.\vec{c}+\vec{c}.\left(\vec{a}+\vec{b}\right)$ $= \lvert\vec{a}\rvert^2+\lvert\vec{b}\rvert^2+\lvert\vec{c}\rvert^2 = 9+16+25=50$ $\lvert\vec{a}+\vec{b}+\vec{c}\rvert = 5\sqrt{2}$	
14	$\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ $\Rightarrow \vec{\beta}_1 = \lambda\vec{\alpha} \Rightarrow \vec{\beta}_1 = \lambda\left(3\hat{i}-\hat{j}\right)$	

	Let $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2 \Rightarrow \vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1 = (2-3\lambda)\hat{i} + (1+\lambda)\hat{j} - 3\hat{k}$ $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha} \Rightarrow \vec{\beta}_2.\vec{\alpha} = 0$ $\therefore 3(2-3\lambda) - (1+\lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$ $\therefore \vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$ And $\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$	
15	$\vert \vec{a}\vert = 3$,$\vert \vec{b}\vert = 5$,$\vert \vec{c}\vert = 7$ $\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow \vec{a} + \vec{b} = -\vec{c}$ $(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = -\vec{c}.-\vec{c} \Rightarrow \vert\vec{a}\vert^2 + 2(\vec{a}.\vec{b}) + \vert\vec{b}\vert^2 = \vert\vec{c}\vert^2 \Rightarrow \vec{a}.\vec{b} = \frac{15}{2}$ $cos\,\theta = \frac{\vec{a}.\vec{b}}{\vert\vec{a}\vert\vert\vec{b}\vert} = \frac{1}{2}$ $\therefore \theta = \frac{\pi}{3}$	
	<u>THREE DIMENSIONAL GEOMETRY</u>	
	<u>MCQ</u>	
1	(a) -2/7, 3/7, 6/7	
2	(b) $\vec{r} = (3\hat{\imath} - 4\hat{\jmath} + 3\hat{k}) + \mu(-5\hat{\imath} + 7\hat{\jmath} + 2\hat{k})$	
3	(c) π/6	
4	(b) 0,0,1	
5	(c) (0,-3,0)	
	<u>2 Marks</u>	
6	The lines are : $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1/k}$ and $\frac{x-2}{1} = \frac{y+1}{1/2} = \frac{z-1}{-1}$ Since these lines are perpendicular $1x1 + -1x\frac{1}{2} + \frac{1}{k}x - 1 = 0$ k = 2	
7	i) Cartesian equation : $\frac{x-3}{2} = \frac{y+2}{-2} = \frac{z+5}{11}$	

	ii) Vector Equation : $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ $\vec{r} = (3\vec{i} - 2\vec{j} - 5\vec{k}) + \lambda(2\hat{i} - 2\hat{j} + 11\vec{k})$	
8	The equation of line is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3}$ Any point on this line is (3k+2, -k+2, -3k+1) 3k+2=4 $\Rightarrow$ k=2/3 z co ordinate = -3x2/3 + 1 =-2 + 1 = -1	
9	Let D(a,b,c) , A (1,2,3) , B(-1,-2,-3) , C(2,3,2) Midpoint of AC = Midpoint of BD (3/2,5/2,5/2) = (-1+a /2, -2+b /2,-3+c /2) a=4, b=7 , c=8 D(4,7,8) Equation of CD is $\frac{x-2}{2} = \frac{y-3}{4} = \frac{z-2}{6}$	
10	Equation of the line through the points A(4,7,8) and B(2,3,4) is $\frac{x-4}{-2} = \frac{y-7}{-4} = \frac{z-8}{-4}$ Equation of the line through the points C(-1,-2,1) and D (1,2,5) is $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z-1}{4}$ Direction ratios of lines AB and CD are proportional. So the two lines are parallel .	
	3 Marks	
11	$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11} \Rightarrow \frac{x-1}{-3} = \frac{y-2}{2\lambda/7} = \frac{z-2}{11/5} \rightarrow (i)$ $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \Rightarrow \frac{x-1}{-\frac{3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} \rightarrow (ii)$ (i) and (ii) are perpendicular $-3(-\frac{3\lambda}{7}) + \frac{2\lambda}{7}(1) + \frac{11}{5}(-5) = 0 \Rightarrow \lambda = 7$	
12	$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$ The two given lines are $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$ $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$ $\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$	

	$\left\|\vec{b}_1 \times \vec{b}_2\right\| = \sqrt{29}$ $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 8$ $d = \left\|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\left\|\vec{b}_1 \times \vec{b}_2\right\|}\right\| = \frac{8}{\sqrt{29}}$	
13	Midpoint of (2,-1,5) and (1,2,3) is $(\frac{3}{2}, \frac{1}{2}, 4)$ Dr of line joining midpoint and (1,2,3) is ½,$\frac{-3}{2}$,1 Equation of the line is $\bar{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(\frac{1}{2}\hat{\imath} - \frac{\hat{3}}{2}\hat{\jmath} + \hat{k})$ Cartesian equation is $\frac{x-1}{1/2} = \frac{y-2}{-3/2} = \frac{z-3}{1}$	
14	Equation of the line through A(0,-1,-1) and B(4,5,1) is $\frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2}$ Equation of the line through C(3,9,4) and D(-4,4,4) is $\frac{x-3}{-7} = \frac{y-9}{-5} = \frac{z-4}{0}$ $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & 10 & 5 \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 0$ Hence the two lines AB and CD intersect each other.	
15	P (–2, 4, – 5) is the given point. Any point Q on the line is given by (3λ-3, 5λ + 4, 6λ-8) $\overrightarrow{PQ} = (3\lambda - 1)\hat{\imath} + 5\lambda\hat{\jmath} + (6\lambda - 3)\hat{k}$ PQ and the given line are perpendicular $\therefore\ (3\lambda - 1)3 + 5\lambda.5 + (6\lambda - 3)6 = 0 \quad \Rightarrow \lambda = \frac{3}{10}$ $\overrightarrow{PQ} = \frac{1}{10}\hat{\imath} + \frac{1}{10}\hat{\jmath} - \frac{12}{10}\hat{k}$ Magnitude of PQ = $\sqrt{\frac{37}{10}}$	
	5 Marks	
16	Let D be the foot of the perpendicular Equation of line BC is $\bar{r}$= -j+3k + λ (2i-2j -4k) Therefore any point D on the line is (2 λ, -1-2 λ, 3- 4 λ) Since AD is perpendicular to BC, (2 λ -1)x2 + (-1-2 λ -8)x(-2) + (3-4 λ -4)x(-4)=0	

	Solving we get λ = -5/6 So the required point D is (-5/3,2/3, 19/3) Let E(a,b,c) be the image of A , then D is the midpoint of AE Using the midpoint formula find image E$(-\frac{13}{3},-\frac{20}{3},\frac{26}{3})$ $\textit{and distance AD can be obtained by using distance formula}$	
17	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda \Rightarrow$ x= $2\lambda+1, y=3\lambda+2, z=4\lambda+3$ $\frac{x-4}{5}=\frac{y-1}{2}=z$ = k $\Rightarrow$ x=5k+4 ,y =2k+1, z=k Lines intersect means $2\lambda+1=5k+4$ and $3\lambda+2=2k+1$ $\lambda=-1 \text{ and } k=-1$ $4\lambda+3=k$ this equation is true for $\lambda=-1 \text{ and } k=-1$ The lines are intersecting and the point of intersection is (-1,-1,-1)	
18	Equation of the line passing through P(-1,3,-2) is $\frac{x+1}{a}=\frac{y-3}{b}=\frac{z+2}{c}$ Since it is perpendicular to the two given lines a+2b+3c =0 and -3a+2b+5c =0 Solving a =2 , b=-7 , c=4 Equation of required line is $\frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$	
19	Equation of line passing through (2,-1,3) is $\frac{x-2}{a}=\frac{y+1}{b}=\frac{z-3}{c}$..........(1) Given: line (1) is perpendicular to the two given lines $\therefore$ 2a-2b+c =0 ....(2) and a+2b+2c=0 ....(3) Solving eq. (2) and (3) a=2 , b=1 , c= -2 Required equation of line is $\vec{r}=\left(2\hat{\imath}-\hat{\jmath}+3\hat{k}\right)+\mu\left(2\hat{\imath}+\hat{\jmath}-2\hat{k}\right)$	
20	Given lines are $\vec{r}=\left(6\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)+\lambda\left(\hat{\imath}-2\hat{\jmath}+2\hat{k}\right)$ and $\vec{r}=\left(-4\hat{\imath}-\hat{k}\right)+\mu\left(3\hat{\imath}-2\hat{\jmath}-2\hat{k}\right)$ Cartesian equation of the two given lines are $\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2}=\lambda$ ....(1) $\Rightarrow P=(\lambda+6,-2\lambda+2,2\lambda+2)$	

$$\frac{x+4}{3}=\frac{y}{-2}=\frac{z+1}{-2}\ldots\ldots\ldots\ldots\ldots\ldots(2)\Rightarrow Q=(3\mu-4,-2\mu,-2\mu-1)$$

Direction ratios of PQ are

$$3\mu-\lambda-10,-2\mu+2\lambda-2,-2\mu-2\lambda-3$$

Direction ratios of line (1) are 1 ,-2 ,2

Direction ratios of line (2) are 3 , -2 , -2

Distance between the two given lines will be shortest if PQ is perpendicular to line (1) as well as to line (2)

$$\therefore 1(3\mu-\lambda-10)-2(-2\mu+2\lambda-2)+2(-2\mu-2\lambda-3)=0\Rightarrow 3\mu-9\lambda-12=0\ldots\ldots\ldots(3)$$

$$3(3\mu-\lambda-10)-2(-2\mu+2\lambda-2)-2(-2\mu-2\lambda-3)=0\Rightarrow 17\mu-3\lambda-20=0\ldots\ldots\ldots(4)$$

Solving (3) and (4)

λ = -1 and μ =1

The points on the lines they should reach so that the distance between them is the shortest is P(5,4,0) and Q(-1,-2,-3)

Shortest distance = PQ = 9

LINEAR PROGRAMING

MCQ QUESTIONS

1 Feasible region is the set of points which satisfy
(a) The objective functions
(b) Some the given constraints
(c) All of the given constraints
(d) None of these

2 The solution set of the inequality $4x + 5y > 6$ is

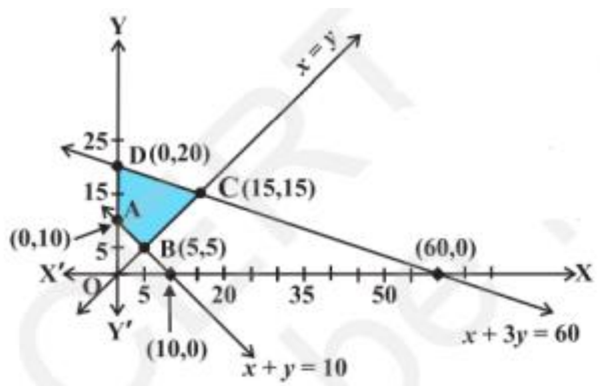
(a) an open half-plane not containing the origin.

(b) an open half-plane containing the origin.

(c) the whole XY-plane not containing the line inequality $4x + 5y = 6$.
(d) a closed half plane containing the origin.

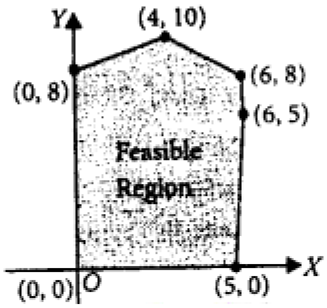
3 Maximize $Z = 10x_1 + 25x_2$, subject to $0 \leq x_1 \leq 3$, $0 \leq x_2 \leq 3$, $x_1 + x_2 \leq 5$
(a) 80 at (3, 2)
(b) 75 at (0, 3)
(c) 30 at (3, 0)
(d) 95 at (2, 3)

4 Based on the given shaded region as the feasible region in the graph, at which point(s) is the objective function $Z = 3x + 9y$ maximum?



a) Point B b) Point C c) Point D d) every point on the line segment CD

5 The feasible, region for an LPP is shown shaded in the figure. Let $Z = 3x – 4y$ be the objective function. A minimum of Z occurs at



(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)

CASE BASED QUESTIONS

1 A train can carry a maximum of 300 passengers. A profit of Rs. 800 is made on each executive class and Rs. 200 is made on each economy class. The IRCTC reserves at least 40 tickets for executive class. However, atleast 3 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class ticket is x and that of economy class ticket is y. Based on the above information, answer the following questions.

(i) The objective function of the LPP is:

(a) Maximise Z = 800x + 200y (b) Maximise Z = 200x + 800y

(c) Minimise Z = 800x + 200y (d) Minimise Z = 200x + 800y

(ii) Which among these is a constraint for this LPP?

(a) $x+y \geq 300$ (b) $y \geq 3x$ (c) $x \leq 40$ (d) $y \leq 3x$

(iii) Which among these is not a corner point for this LPP?

(a) (40,120) (b) (40, 260) (c) (30, 90) (d) (75, 225)

(iv) The maximum profit is:

(a) Rs.56000 (b) Rs. 8400 (c) Rs. 205000 (d) Rs. 105000)

(v) Which corner point the objective function has minimum value?

(a) (40,120) (b) (40, 260) (c) (30, 90) (d) (75, 225)

2. An aeroplane can carry a maximum of 200 passengers. A profit of Rs.1000 is made on each executive class ticket and a profit of Rs.600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. It is given that the number of executive class tickets is x and that of y economy class tickets is y

1 Find the maximum value of $x+y$

(a) 250 (b) 200 (c) 300 (d) 100

2 Calculate maximum profit

(a) 136000 (b) 163000 (c) 156000 (d) 165000

3 At which point profit is maximum

(a) (20,180) (b) (20,80) (c) (40,160) **(d) (20,40)**

3. A manufacturing company makes two models X and Y of a product. Each piece of model X requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model Y requires 12 labour hours of fabricating and 3 labour hours for finishing, the maximum labour hours available for fabricating and finishing are

180 and 30 respectively. The company makes a profit of Rs. 8000 on each piece of model X and Rs. 12000 on each piece of model Y. Assume x is the number of pieces of model X and y is the number of pieces of model Y.

1 Which among these is not a constraint for this LPP?

(a) $9x+12y\geq180$ (b) $3x+4y\leq60$ (c) $x+3y\leq30$ (d) None of these

2 The shape formed by the common feasible region is:

(a) Triangle (b) Quadrilateral (c) Pentagon (d) hexagon

3 Which among these is a corner point for this LPP?

(a) (0,20) (b) (6,12) (c) (12,6) (d) (10,0)

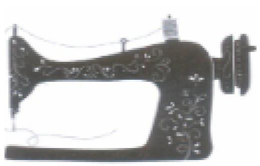
4 Maximum of Z occurs at

(a) (0,20) (b) (0,10) (c) (20,10) (d) (12,6)

5 The sum of maximum value of Z is:

(a) 168000 (b) 160000 (c) 120000 (d) 180000

4 Suppose a dealer in rural area wishes to purchase a number of sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items for storage. An electronic sewing machine costs him Rs. 360 and a manually operated sewing machine Rs.240. He can sell an electronic sewing machine at a profit of Rs. 22 and a manually operated sewing machine at a profit of Rs.18. Based on the above information, answer the following questions.



(i) Let x and y denote the number of electronic sewing machines and manually operated sewing machines purchased by the dealer. If it is assumed that the dealer purchased atleast one of the given machines then:

(a) $x+y\geq 0$ (b) $x+y< 0$ (c) $x+y> 0$ (d) $x+y\leq 0$

(ii) Let the constraints in the given problem is represented by the following inequalities:

$x+y\leq 20$; $360x+240y\leq 5760$ and $x, y\geq 0$. Then which of the following point lie in its feasible region.

(a) (0,24) (b) (8,12) (c) (20,2) (d) None of these

(iii) If the objective function of the given problem is maximize $Z = 22x+18y$, then its optimal value occur at:

(a) (0,0) (b) (16,0) (c) (8,12) (d) (0,2)

(iv) Suppose the following shaded region APDO, represent the feasible region corresponding to mathematical formulation of the given problem.

Then which of the following represent the coordinates of one of its corner points.

(a) (0,24) (b) (12,8) (c) (8,12) (d) (6,14)

(v) If an LPP admits optimal solution at two consecutive vertices of a feasible region, then

(a) The required optimal solutionis at a mid pointof the line joining two points.

(b) The optimal solution occurs at every point on the line joining these two points.

(c) The LPP under consideration is not solvable.

(d) The LPP under consideration must be reconstructed.

5 Corner points of the feasible region for an LPP are (0, 3), (5, 0), (6, 8), (0, 8).
Let $Z = 4x - 6y$ be the objective function.

Based on the above information, answer the following questions.

(i) The minimum value of Z occurs at

(a) (6,8) (b) (5,0) (c) (0,3) (d) (0,8)

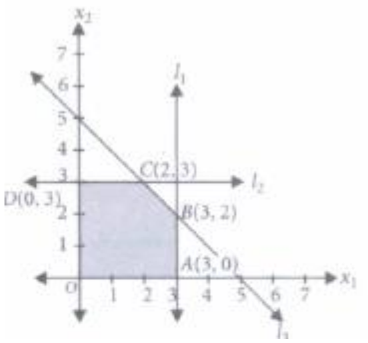
(ii) Maximum value of Z occurs at

(a) (5,0) (b) (0,8) (c) (0,3) (d) (6,8)

(iii) Maximum of Z - Minimumof Z =

(a) 58 (b) 88 (c) 78 (d) 68

(iv) The corner points of the feasible region determined by the system of linear inequalities



(a) (0,0), (-3,0),(3,2), (2,3)   (b) (3,0),(3,2),(2,3),(0,-3)

(c) (0,0),(3,0),(3,2),(2,3),(0,3)   (d) None of these

(v) The feasible solution of LPP belongs to

(a),first and second quadrant   (b) first and third quadrant

(c)only second quadrant   (d) only first quadrant

PROBABILITY

MCQ

1 If P (A) = 0.8, P (B) = 0.5 and P (B|A) = 0.4, what is the value of P (A ∩ B)?

A. 0.32   B. 0.25   C. 0.1   D. 0.5

2 If P(A) = 0.4, P(B) = 0.7 and P(B/A) = 0.6. Find P(A ∪ B).

A. 0.46   B. 0.86   C. 0.76   D. 0.54

3 The probability of solving the specific problems independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that exactly one of them solves the problem.

A. 1   B. ½   C. ⅓   D. ¼

4 The probability of obtaining an even prime number on each die,

when a pair of dice is rolled is

(a) 0 (b) 1/3 (c) 1/12 (d) 1/36

5 If P(A) = 4/5 P((A ∩ B) = 7/10 find P(B/A)

(a)) 7/8 (b) 1/8 (c)1/10 (d) 17/30

VERY SHOERT ANSWER (2 MARKS EACH)

1 An unbiased coin is tossed 4 times.Find the probability of getting atleast one head

2 Two independent events A and B are given such that P(A) = 0.3, P(B) = 0.6,

find P(A and not B)

3 The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, evaluate P($\bar{A}$) + P($\bar{B}$)

4 The probability distribution of X is

X	0	1	2	3
P(X)	0.2	k	k	2 k

Evaluate the value of k

5 An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1 Evaluate P(A ∪ B), if 2P (A) = P(B) = 5/13 and P(A/ B) = 2/5.

2 A couple has 2 children. Find the probability that both are boys, if it is known that one of them is a boy.

5 Find the probability distribution of getting number of heads while tossing three coins together

6 A random variable X has following distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2K^2$	$7K^2+k$

Determine (i) k (ii) $P(X<3)$ (iii) $P(X>6)$

7 There are 4 cards numbered 1 to 4,one number on one card.Two cards are drawn at random without replacement.Let X denote the sum of the numbers on the two drawn cards. Find the mean of X

CASE BASED QUESTIONS

1 In a play zone, Aastha is playing crane game. It has 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If Aastha draws two balls one after the other without replacement, then answer the following questions.

(i) What is the probability that the first ball is blue and the second ball is green?

(a)5/119 (b) 12/119 (c) 6/119 (d) 7/119

(ii) What is the probability that the first ball is yellow and the second ball is red?

(a) 6/119 (b) 8/119 (c) 24/119 (d) none of these

(iii) What is the probability that both the balls are red?

(a) 4/85 (b) 204/595 (c) 12/119 (d) 64/119

(iv) What is the probability that the first ball is green and the second ball is not yellow?

(a) 10/119 (b) 6/85 (c) 12/119 (d) none of these

(v) What is the probability that both the balls are not blue?

(a) 6/595 (b) 12/85 (c) 15/595 (d) 253/595

2 A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2, 0.1 and 0.4. The probabilities that he will be late are 0.25, 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.

Based on the above information, answer the following questions.

(i) When the doctor arrives late, what is the probability that he comes by metro?

(a) 5/4 (b) 2/7 (c) 5/21 (d) 1/6

(ii) When the doctor arrives late, what is the probability that he comes by cab?

(a) 4/21 (b) 1/7 (c) 5/14 (d) 2/21

(iii) When the doctor arrives late, what is the probability that he comes by bike?

(a) 5/21 (b)4/7 (c) 5/6 (d) 1/6

(iv) When the doctor arrives late, what is the probability that he comes by other means of transport?

(a) 6/7 (b) 5/14 (c) 4/21 (d) 2/7

(v) What is the probability that the doctor is late by any means?

(a) 4/21 (b) 0 (c) ½ (d) ¼

3 On a holiday, a father gave a puzzle from a newspaper to his son Ravi and his daughter Priya. The probability of solving this specific puzzle independently by Ravi and Priya are 1/4 and 1/5 respectively.

Based on the above information, answer the following questions.

(i) The chance that both Ravi and Priya solved the puzzle, is

(a) 10% (b) 5% (c) 25% (d) 20%

(ii) Probability that puzzle is solved by Ravi but not by Priya, is

(a) ½ (b) 1/5 (c) 3/5 (d)1/3

(iii) Find the probability that puzzle is solved.

(a) ½ (b) 1/5 (c) 2/5 (d) 5/6

(iv) Probability that exactly one of them solved the puzzle, is

(a) 1/30 (b) 1/20 (c) 7/20 (d) 3/20

(v) Probability that none of them solved the puzzle, is

(a) 1/5 (b) 3/5 (c) 2/5 (d) None of these

4 One day, a sangeet mahotsav is to be organised in an open area of Rajasthan. In recent years, it has rained only 6 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he incorrectly forecasts rain 20% of the time.

If leap year is considered, then answer the following questions

(i) The probability that it rains on chosen day is

(a) 1/366 (b) 1/73 (c) 1/60 (d) 1/61

(ii) The probability that it does not rain on chosen day is

(a) 1/366 (b)5/366 (c) 360/366 (d)None of these

(iii) probability that the weatherman predicts correctly is

(a)5/6 (b) 7/8 (c) 4/5 (d)1/5

(iv) The probability that it will rain on the chosen day, if weatherman predict rain for

that day, is

(a) 0.0625 (b) 0.84 (c) 0.74 (d) 0.64

5 To teach the application of probability a maths teacher arranged a surprise game for of his students namely Archit, Aadya, Mivaan, Deepak and Vrinda. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers. Based on the above information, answer the following questions

(i) Teacher ask Vrinda, what is the probability that both tickets drawn by Archit shows even number?

(a) 1/50 (b) 12/49 (c) 13/49 (d) 15/49

(ii) Teacher ask Mivaan, what is the probability that both tickets drawn by Aadya shows odd number?

(a) 1/50 (b) 2/49 (c) 12/49 (d) 5/49

(iii)Teacher ask Deepak, what is the probability that tickets drawn by Mivaan, shows a multiple of 4 on one ticket and a multiple 5 on other ticket?

(a)14/245 (b)16/245 (c) 24/245 (d)36/245

(iv) Teacher ask Archit, what is the probability that tickets are drawn by Deepak, shows a prime number on one ticket and a multiple of 4 on other ticket?

(a)3/245 (b) 17/245 (c) 18/245 (d) 36/245

(v) Teacher ask Aadya, what is the probability that tickets drawn by Vrinda, shows an even number on first ticket and an odd' number on second ticket?

(a) 15/98 (b)25/98 (c) 35/98 (d) None of these

ANSWERS

LINEAR PROGRAMING

1 . C 2. A 3.D 4.D 5.B

CASE BASED QUESTIONS

1 (i) a (ii) b (iii) (c)(30,90) (iv) (d)1050000 (v) (a)(40,120)

2 .(i) b (ii) a (iii) c

3 I) A 2) B 3) C 4) D 5) A

4 I)C 2) B 3) C 4) C 5) B

5 (i) d (ii) a (iii) d (iv) c (v) d

PROBABILITY

MCQ (1 marks each)

1 A 2.B 3.B 4 .D 5 .A

VERY SHORT ANSWER (2 MARKS EACH)

1 P(getting atleast one head) = 1– P(no head)

$= 1 - P(\text{all tails})$

$= 1 - \left(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\right)$

$= 1 - \frac{1}{16} = \frac{15}{16}$

2 P(A and not B) $= P(A \cap B') = P(A) - P(A \cap B)$

$= 0.3 - 0.18$ $\quad (P(A \cap B) = P(A) \times P(B))$

$= 0.12$

3 Given $P(A \cup B) = 0.6$, $(P(A \cap B) = 0.3$

$P(A \cup B) = P(A) + P(B) - (P(A \cap B)$

$P(A) + P(B) = 0.9$

$(1 - P(\bar{A})) + (1 - P(\bar{B})) = 0.9$

$P(\bar{A}) + P(\bar{B}) = 2 - 0.9 = 1.1$

4 $\sum P(X) = 1$

$$0.2 + 4\,k = 1$$

k = 0.2

5 probability (both drawn balls are black)= 10/15 x 9/14= 3/7

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

We have, $2P(A) = P(B) = \frac{5}{13}$

$\Rightarrow P(A) = \frac{5}{26}$ and $P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

$\because P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\therefore \quad \frac{2}{5} = \frac{P(A \cap B)}{5/13}$

$\Rightarrow \quad P(A \cap B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5+10-4}{26} = \frac{11}{26}$

:

2 let G and g represent older and younger girl child. The sample space of the given question is S = {BB, BG, GB, GG}

∴ n(S) = 4

Let A be the event that both children are boys.

Then, A = {BB} then P(A) = ¼

Given B = {BB, BG,GB} P(B) = 3/4

A∩ B= {BB} P(A∩ B) = ¼

P(A/B) = 1/3

3 S={ HHH,HTH,HHT,THH,HTT,THT,TTH,TTT}

X denotes number of heads, then X takes values 0,1,2,3

P(x=0) =1/8 P(X=1) = 3/8 P(X=2) = 3/8 P(X=3) = 1/8

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

4 $\sum P(X) = 1$

$10\,k^2 + 9k - 1 = 0$

K = -1   k= 1/10

(i) Since k can not be negative   k = 1/10

(ii) P(X<3) = 3/10   (iii) P(X>6) = 17/100

5   S = { (1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(4,1),(4,2),(4,3),(3,1),(3,2),(3,4)}

X denotes sum of numbers ,   X takes values 3,4,5,6,7

P(X=3) =2/12   P(X=4) = 2/12   P(X=5)= 4/12   P(X=6)=2/12   P(X=7) =2/12

X	3	4	5	6	7
P(X)	2/12	2/12	4/12	2/12	2/12

$\sum x_i p_i = \frac{6}{12}+\frac{8}{12}+\frac{20}{12}+\frac{12}{12}+\frac{14}{12}$

$=5$

CASE BASED QUESTIONS

1   (i) c   (ii) b   (iii) a   (iv) c   (v) d

2   (i) b   (ii) c   (iii) d   (iv) c   (v) a

3   (i) b   (ii) b   (iii) c   (iv) c   (v) b

4   (i) d   (ii) c   (iii) c   (iv) a   (v) a

5   (i) b   (ii) c   (iii) c   (iv) d   (v) b